Frictional energy dissipation for coupled systems subjected to harmonically varying loads

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ABSTRACT
Energy dissipation due to frictional slip under periodic loading in the presence of frictional coupling is investigated. The effects on energy dissipation of the relative phase between periodic loads along the normal and tangential directions are explored. The results show that, with increasing frictional coupling, the relative phase of the periodic loads approaches \( \pi/2 \) when maximum dissipation occurs. This behaviour is comparable to the uncoupled frictional case, for which maximum dissipation occurs at the relative phase of \( 0.6\pi \). Analytical frictional dissipation is obtained for in the case of no relative phase between the loads.

1. Introduction

Periodic loading due to a vibration environment generally affects many engineering systems, especially frictional systems. The frictional energy associated with the frictional force and slip amounts dissipates the entire energy of the system, resulting in deterioration of the system performance. Examples of adverse effects can be found in the fretting corrosion of electrical connectors, which usually combines fretting wear and corrosion, such as oxidation, under oscillatory motion of small amplitude at the contact interface [1,2] and the fretting fatigue under the action of centrifugal loading and vibration at the root of turbine blades [3].

The effects of periodic loading on frictional systems have already been investigated, leading to several important conclusions: 1) Steady-state frictional contact behavior for uncoupled system exists, regardless of the initial condition [4]; therefore, the normal reactions are not affected by the tangential displacements. For the coupled frictional contact system under cyclic loading, the response as well as frictional dissipation will generally be affected by the initial conditions, leading to fretting damage [5-8]. 2) The relative phase between the loads is closely related to the energy dissipation and transient behaviors of the system [6,9,10].

Previous studies, which focused on the effect of the relative phase on frictional dissipation, were performed considering an uncoupled frictional system with normal and tangential displacements [9,10]. These studies showed that when the relative phase of the tangential and normal loads is \( 0.6\pi \), the system experiences maximum dissipation. These works are meaningful because the model is simple and the steady states are unique in uncoupled systems. However, in coupled frictional systems, the behavior is much more complex, and may not result in a unique steady state. This frictional behavior of elastic systems is affected by elastic coupling for normal and tangential stiffness. In addition, for the coupled multinode contact system under cyclic loading, the initial conditions affect the response as well as frictional dissipation. However, at a stick node in this multi-node system, the position of the node can be identified by the previous history of loading [5]. Thus, if there is only one node, either it never slips and the frictional energy dissipation is zero, or it slips during the steady state, in which case the permanently-stuck nodes are a null set. In other words, the system is not affected by the history of loading for the one-node case, unless it is possible for the system to either slip or never slip, in which case these two states are distinct, rather than forming a continuum. Thus, it is worth exploring the coupled frictional system of a single node with two degrees of freedom, even though the coupled system has at least one additional parameter compared with the uncoupled system, thereby making it impossible to present complete results covering all possible parameter values for this case.

In this study, we extend the previous uncoupled frictional analysis to the coupled case to identify the effect of the relative phase between the loads and coupling on the frictional energy dissipation under the...
Under the quasi-static assumption, the equilibrium equations for the system to the normal and tangential direction can be obtained as
\[ Q = F + k_{11}u + k_{12}v \] (1)
\[ P = N + k_{21}u + k_{22}v. \] (2)

The diagonal stiffnesses of the linear springs \( k_{11}, k_{22} \) are positive and the off-diagonal stiffness should be \( k_{12} = k_{21} \) owing to Maxwell's Reciprocal Theorem. The stiffness matrix composed of \( k_{11}, k_{22}, \) and \( k_{12} \) is positive definite, ensuring that \( k_{11}k_{22} > k_{12}^2 \).

The periodic loading for the system can be written as
\[ N(t) = N_0 + N_1 \sin(\omega t); \quad F(t) = F_0 + F_1 \sin(\omega t + \phi). \] (3)

The Coulomb friction law allows the system to be in one of four states i.e., stick, separation, forward slip, and backward slip. The stick condition holds when the normal force is positive \((N > 0)\), the tangential force is of the form \(|F| \leq \mu N\), and the vertical and tangential displacements are \(v = 0\) and \(u = 0\), respectively. Separation occurs when the vertical displacements are positive \((v > 0)\) and there are no forces \((N = F = 0)\). The forward and backward slip conditions are \(v = 0\), \(u > 0\), \(N \geq 0\), \(F = -\mu N\) and \(u < 0\), \(N \geq 0\), \(F = \mu N\), respectively. Note that the time derivative are represented as dots. The total energy dissipation during one complete cycle of loading is calculated as
\[ W = -\int Q(t)\ddot{u}(t)dt. \] (4)

From Klarbring’s P-matrix criterion \([11]\), the rate contact problem is well-posed when \(k_{11} \pm \mu k_{12} > 0\), showing a critical coefficient of friction \(\mu_c = k_{11}/k_{12}\). If the coefficient of friction is above the critical, the P-matrix criterion fails, resulting in an ill-posed rate contact problem.

2.1. Normalization

The formulation of the coupled frictional system involves many parameters and can be reduced by normalization. Using these dimensionless quantities,
\[ \tilde{N} = \frac{N}{N_0}; \quad \tilde{P} = \frac{P}{N_0}; \quad \tilde{Q} = \frac{Q}{\mu N_0}; \quad \tilde{u} = \frac{k_{11}}{F_1}u; \quad \tilde{v} = \frac{k_{12}}{F_1}v; \quad \tilde{W} = \frac{k_{11}W}{\mu N_0 F_1}. \] (5)

The frictional energy dissipation in this system is not affected by the normal stiffness \(k_{22}\) because the normal stiffness only affects the distance the mass moves vertically during separation periods and no dissipation occurs then. The system is restricted to the case where there is always in contact, meaning that \(N_0 > 0\). In addition, the tangential loading \(F_1\) only affects the mean tangential displacement under cyclic loading, resulting in no dissipation. Thus we can assume to be zero

The normalized equations for equation (2) can be obtained as
\[ \tilde{Q} = \tilde{F} + \tilde{k}_1 \tilde{u} + \tilde{F}_1 \tilde{k}_2 \tilde{v} \] (6)
\[ \tilde{P} = \tilde{N} + \mu \tilde{F}_1 \tilde{k}_1 \tilde{u} + \mu \tilde{F}_1 \tilde{k}_2 \tilde{v}. \] (7)

The normalized total frictional energy dissipation per cycle takes the form
\[ \tilde{W} = -\int \tilde{Q} \ddot{u} dt. \] (8)

The five dimensionless parameters \(\tilde{N}_i, \tilde{F}_i, \phi, \mu, \) and \(k_{12}/k_{11}, \) are effective to determine the dissipation and the results can be obtained by using four parameters if we set \(\mu k_{12}/k_{11}\) which is a measure of coupling in the sense of Klarbring’s P-matrix criterion \([11]\). In order for this criterion to be satisfied, \(\mu k_{12}/k_{11}\) should be less than one. Compared with the uncoupled frictional system, one additional parameter is introduced

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in this case; therefore, it becomes impossible to present the complete results covering all possible parameter values.

2.2. Simulation for the frictional contact

The change in the contact status is simulated using the algorithm of Ahn and Barber [12], which estimates the state by using the criterion of the Coulomb friction law, as shown in equation (9), until the previous identified state does not change. However, the Ahn and Barber algorithm is not appropriate if the limiting friction is reached, specifically when the separation state is changed to stick or slip. These features have been already reported by Cho and Barber [13], suggesting an inelastic impact condition that highlights the effects of elastic recovery. When slip occurs after separation, a normal impulse and the corresponding proportional frictional impulse should be considered to decrease the approach velocity to zero, thus resulting in a stick state. These are summarized as

1. if \( \ddot{v} < 0 \) and \( \ddot{u} > -\mu \dot{v} \), the state changes to forward slip and set \( \dot{v} = 0 \) and \( \ddot{u} = \ddot{u} + \mu \dot{v} \)
2. if \( \ddot{v} < 0 \) and \( \mu \dot{v} < \ddot{u} < -\mu \dot{v} \), the state changes to stick and set \( \dot{v} = \dot{v} = 0 \)
3. if \( \ddot{v} < 0 \) and \( \ddot{u} < \mu \dot{v} \), the state changes to backward slip and set \( \dot{v} = \dot{v} = 0 \) and \( \ddot{u} = \ddot{u} - \mu \dot{v} \)

3. Results

3.1. Frictional energy dissipation for in-phase loading

When the relative phase is zero, the analytical frictional energy dissipation can be obtained by using a similar approach to that proposed by Jang and Barber [10]. Unlike the uncoupled frictional system, the coupled frictional system is affected by the coupling stiffness \( k_{12} \); thus, the conditions for the dissipation can be divided according to the ratio of \( k_{12}/k_1 \).

\[
\dot{W} = 0: |\dot{N}_1| < \frac{-1}{\mu k_r}; |\dot{F}_1| < \mu k_r |\dot{N}_1| + 1
\]

\[
= \frac{4(\dot{F}_1 - \mu k_r |\dot{N}_1| - 1)(\dot{F}_1 - \mu k_r |\dot{N}_1| - (\mu k_r |\dot{F}_1| - |\dot{N}_1|)^2}{(1 - \mu^2 k_1^2)(\dot{F}_1^2 - |\dot{N}_1|^2)}
\]

\[
: |\dot{N}_1| < \frac{-1}{\mu k_r}; \mu k_r |\dot{N}_1| + 1 < |\dot{F}_1| < \frac{1}{\mu k_r} (|\dot{N}_1| + 1)
\]

\[
= \frac{(|\dot{N}_1| + |\dot{F}_1|)(1 - |\dot{N}_1| + \mu k_r |\dot{F}_1|)^2}{(1 + \mu k_r^2)(|\dot{N}_1| + |\dot{F}_1|)(|\dot{N}_1| + \mu k_r |\dot{F}_1|)}
\]

\[
: |\dot{N}_1| < \frac{-1}{\mu k_r}; |\dot{F}_1| > \frac{1}{\mu k_r} (|\dot{N}_1| + 1)
\]

Fig. 3. Contour plots of \( \dot{W} \) with respect to \( \dot{N}_1, \dot{F}_1 \) for \( \mu k_r \), when \( \phi = 0 \).

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where

\[ k_r = \frac{k_{r1}}{k_{r1}} \]

Fig. 2 shows the regions of frictional energy dissipation, which can

Fig. 4. Contour plots of \( \hat{W} \) with respect to \( \hat{N}_1, \hat{F}_1 \) for \( \phi \) and \( \mu k_r \).

Fig. 5. Dimensionless dissipation \( \hat{W} \) with respect to \( \phi \) for various values of \( \mu k_r \) when \( \hat{N}_1 = \hat{F}_1 = 5. \)
be categorized according to the behavior of the coupled frictional system. When $|F_1| > \frac{1}{\mu_k}(|N_1| - 1)$ and $F_1 < \mu_k|N_1| + 1$, which represents the region of (D), the system undergoes shakedown, implying that the system reaches a stick state after an initial period of slip under no separation. If the loading is in the ranges of $|F_1| < \frac{1}{\mu_k}(|N_1| - 1)$ and $F_1 > |N_1|$, which represents the region of (E), the system alternates between stick and separation during periods, resulting in no dissipation.

The regions represented as (A) and (C) in this figure show that the system experiences forward or backward slip and separation occurs during part of the loading cycle. It can be divided by the order of the contact states; for example, the order of the contact states of the cycle for region (A) is stick $\rightarrow$ backward slip $\rightarrow$ separation $\rightarrow$ forward slip $\rightarrow$ stick, and the order of contact states for region (C) is separation $\rightarrow$ backward slip $\rightarrow$ stick $\rightarrow$ forward slip $\rightarrow$ separation. Region (B) shows that the system experiences slip under the contact state without separation during the loading cycle. Compared with regions (A) and (C), a region of higher dissipation is formed. This is confirmed by the contour plot of the dissipation, as shown in Fig. 3. The contour plot also shows the effects of the coupling stiffness $\mu_k$, on the frictional energy dissipation when $\phi = 0$. When the coefficient of friction approaches zero, or the coupling stiffness of $k$ becomes zero, the lines of (3) and (4) approach $|F_1| = |N|$ and $|\dot{F}_1| = 1$, which are the results for the uncoupled frictional system [10]. When the coefficient of friction approaches the critical coefficient of friction 1/$k_c$, the intersection point $N_1'$ becomes infinite and the line of (4) in Fig. 2 approaches $|\dot{F}_1| = |N_1' + 1$, thereby enlarging the dissipation region of (B).

### 3.2. Dissipation for out-of-phase loading

Contour plots of the dimensionless frictional energy dissipation $\tilde{W}$ with respect to $N_1$, $\tilde{F}_1$ for several relative phases $\phi$ and $\mu_k$, are shown in Fig. 4. The previous work regarding the effect of the relative phase on frictional energy dissipation for the uncoupled system already showed that the dissipation reaches a maximum when the relative phase is $0.6\pi$ [9,10]. For a coupled system, the frictional energy dissipation is maximum when the relative phase is $\pi/2$, regardless of $\mu_k$.

As the coupling stiffness $\mu_k$, increases, the region of no-dissipation enlarges when phase lag is included in $\phi < \pi/2$ and reduces when $\phi > \pi/2$. The shape of the no-dissipation region for $\phi < \pi/2$ follows the limiting lines of (2) and (4) in Fig. 2. For the range of $\phi > \pi/2$, the circular and rectangular shape of the no-dissipation region for $\phi = 0$ changes to triangular as $\mu_k$ increases.

The frictional energy dissipation according to the combination of the coupling stiffness and the frictional coefficient is specifically shown in Fig. 5.

If $N_1 < 1$, the full contact region occurs and the relative phase does not significantly affect dissipation as $\tilde{F}_1$ increases, except when $\tilde{F}_1$ is small. This behavior is similar to the case of the uncoupled contact. If $N_1 > 1$, a region of $\tilde{N}_1$ and $\tilde{F}_1$ with high dissipation is formed as the relative phase increases, as shown in the contour plots of Fig. 4. The maximum dissipation occurring at a certain $\phi$ for different coupling effects $\mu_k$, is shown in Fig. 5. For the uncoupled case, a maximum dissipation near $\phi = 0.6\pi$ occurs. As the coupling effect increases, the maximum dissipation occurs at $\phi = \pi/2$.

The effects of $\mu_k$, on the frictional energy dissipation can easily be understood in Figs. 6 and 7. Fig. 6 shows the dimensionless dissipation $\tilde{W}$ with respect to $\mu_k$, for $N_1 = 0.5$ and several values of $\tilde{F}_1$ when (a) $\phi = 0$ and (b) $\phi = \pi/2$. Specifically, when $\phi = 0$, the dissipation decreases monotonically as $\mu_k$ increases for $\tilde{F}_1$ up to 1.5. Above $\tilde{F}_1 = 1.5$, the dissipation increases and then decreases after a certain value of $\mu_k$.

When the phase lag is $\pi/2$, the dissipation pattern is similar to the case of Fig. 6 (a) above $\tilde{F}_1 = 1.5$. Fig. 7 shows the dimensionless dissipation $\tilde{W}$ with respect to $\mu_k$, for $N_1 = 0.5$ and several values of $\tilde{F}_1$ when (a) $\phi = 0$ and (b) $\phi = \pi/2$. The overall dissipation trend according to the variation of $\mu_k$, is similar to that shown in Fig. 6.

One of extreme case of the problem is that the coefficient of friction asymptotically approaches infinity, which is regarded as “relaxation
damping” [14]. During the infinite friction process, stick remains until the normal force is zero, and at this point the spring can have strain energy which will be dynamically released when separation occurs. This will be dissipated as damped vibration, so with sufficiently big ratio between natural frequency and loading frequency, the system will lose the strain energy in the spring just before separation in each cycle. Thus, there is no energy loss in stick.

4. Conclusions

In this study, we investigated the energy dissipation considering the effect of the coupling stiffness with a simple coupled frictional contact system under periodic loading. For this, we calculated the amount of analytical frictional energy dissipation for in-phase loads, and found that a higher-energy dissipation region appeared due to the coupling effect. Further, the numerical experiments showed that the frictional energy dissipation reached the maximum value at the phase angle $\phi = \pi/2$ as the frictional coupling stiffness approached the critical limiting coefficient.

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