Frictional contact behaviors between beam and cylinder under cyclic loading

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1. Introduction

Facilities in nuclear power plants are sensitive to problems related to friction-induced vibration. An example of such a situation is the fuel rod supports in pressurized water reactors. The fuel rods are supported by a spacer grid cell that consists of springs or dimples of low stiffness [1]. Owing to structural limitations, the contact interfaces between the fuel rods and spacer grid are vulnerable to cyclic relative displacements under vibration, thus leading to grid-to-rod fretting (GTRF). This fretting wear is one of the main causes of fuel leakage [2–3]. Many studies have been performed to investigate fretting wear damage [4], the effect of the amplitude and frequency of the excitation force on this fretting [5], and the design issues involved in improving the mechanical/structural performance of the components undergoing this type of damage [6]. The results demonstrate that it is necessary to understand in detail the contact behavior in the fuel rod support due to the fretting load.

A spacer grid cell consists of a flexural element that can be considered to be a beam. Thus, the contact model between the spacer grid cell and fuel rod can be idealized by considering it to be a contact model between a beam and cylinder. This beam and cylinder contact model has already been studied using a higher order beam theory, Fourier transform technique, and finite element analysis [7–9]. These studies were conducted mainly using models that were assumed to have frictionless contacts. The main feature of the beam and cylinder frictionless contact problem is that when the applied force is small, there is a Hertzian stress distribution in the beam. When the force induces a larger contact area as compared to the beam thickness, a finite strip of contact is developed and the non-zero tractions are limited to a pair of concentrated forces at the two edges of this segment, thus resulting in four-point bending. In addition, the extent of the contact region is a fixed ratio of the beam thickness, which is independent of the concentrated load that is predicted by beam theory, and the corresponding distribution of contact pressure in this contact region has a universal form [9].

However, contact behavior under periodic loading, while considering the friction between the beam and cylinder, is rarely investigated. Many studies regarding the contact problem under periodic loading are focused on the Hertzian contact between a semi-infinite plate and cylinder [10] or the receding contact between a semi-infinite plate and finite strip indenter [11]. The majority of these frictional behavior studies [12–19] involve the investigation of the frictional contact behavior under normal periodic loading and have presented the contact state change due to loading and unloading, the contact behavior change during the loading cycle, and the variation in the frictional energy dissipation according to the load amplitude. However, in the beam and cylinder contact problem in this study, a bending deformation occurs in the beam due to the load acting on the cylinder, which can simultaneously induce a relative sliding displacement and rolling displacement between the contact surfaces. Furthermore, the effect of periodic loading on the beam and cylinder contact is considered to be a new research topic that is rarely discussed. The main purpose of this study is to investigate the various contact patterns that may be observed between the beam and cylinder structure in a frictional contact problem.

In this study, we shall investigate the frictional contact between a beam and rigid cylinder under periodic loading using finite element
analysis. The variation in the contact status and corresponding contact traction under periodic loading is thoroughly examined.

2. Description of a numerical model

The numerical model considered in this study includes a simply supported beam of length \( L \) and depth \( h \), which contacts with a rigid cylinder of radius \( R \) at the mid-point of the beam owing to a force \( P \) per unit length, as shown in Fig. 1. The radius of the cylinder \( R \) is greater than 2000\( h \). A frictional coefficient \( f \) of 0.2 is prescribed at the contact interfaces [20]. This simply supported beam has two different support conditions at its two ends, i.e., one end is pinned and the other is free to roll. The simply supported beam is non-symmetric because the horizontal displacements are affected by two different support conditions. In order to explain the effect of the support conditions on the contact behavior, we construct two symmetric models that consider a pinned and roller support, respectively, as shown in Fig. 2. In the following results, the nondimensional variable \( \xi = x/(L/2) \) is used.

The finite element model of the problem has sufficiently refined meshes that have more than 150 contact nodes at any contact region, and it comprises square meshes of \( 0.02 \times 0.02 \) mm as shown in Fig. 1.

The model has 127,551 four-node bilinear plane-strain quadrilateral elements, with a total of 125,000 nodes with \( h/L = 0.02 \). The contact algorithm used for this plane-strain model is the penalty method in the ABAQUS/standard program. The static solver is used with a fixed increment of 0.01 to simulate a quasi-static analysis. The material of the beam is elastic and requires the modulus of elasticity and Poisson’s ratio,
and the cylinder is treated as a rigid body with respect to a reference point. The load is prescribed at this reference point using the AMPLITUDE option. The boundary condition for the roller support at the end of the beam is considered to be that of a spring with a spring constant of 0.001 (N/mm). In the case of the pinned support, the node at the end of the beam is constrained to the y-axis.

3. Results

3.1. Beam contact problem with a pinned support under monotonic loading

Before exploring the contact behaviors of the beam and cylinder interaction due to cyclic loading, we shall investigate the contact behavior due to monotonic loading. The advantage of this analysis is the categorization of the loading range, which may result in various contact situations. When a load $P$ is applied to a rigid cylinder, the rigid cylinder and beam contact at the center of the beam. If the load is small, a single contact area appears at the center of the beam. When the applied load becomes greater than $P_0$, the contact area separates into two contact regions, and this separation process is caused by the difference in curvature between the beam and cylinder [9]. Thus, $P_0$ is defined as the load that separates the contact area into two contact regions. This is illustrated in Fig. 3, which shows the variation of the contact pressure according to the applied load. When $P/P_0 = 0.57$, which is the load ratio for the frictionless contact problem of a beam and cylinder, is obtained by using $4EI/R$, where $EI$ is the bending rigidity of the beam, the contact area is located in the middle of the beam. When the load increases such that the ratio of $P/P_0$ is 1.00, the separation of the contact area is noticeable.

For a pinned support, the frictional beam contact according to the applied load can be described in three different ranges of the applied load. When the applied load is in the range of $0 < P/P_0 < 1$, the contact region is predominantly a stick region. The contact status can be confirmed from Fig. 4(a), which shows the normalized contact pressure and contact shear distribution on the right side of the beam with respect to the symmetric axis. The normalized contact pressure and contact shear are defined as $bfp(ξ)/P_0$ and $bq(ξ)/P_0$, respectively, where $b$ is the contact semi-width. The contact shear distribution is different from the general frictional Hertzian contact traction distribution, which is generally known as the “micro-slip” phenomenon [10]. If the load is in the range of $1 < P/P_0 < 1.88$, there are two contact areas and they are fully stuck. Fig. 4(b) shows the normalized contact pressure and contact shear distribution for the separated contact area at the right side of the beam. Finally, if the load is greater than 1.88$P_0$, some micro-slip starts to appear, as shown in Fig. 4(c). These variations in the contact status during a monotonic loading can be easily observed in Fig. 5. For a load greater than $P_0$, the distribution of the contact pressure spreads over a greater area of the contact region. As the load increases beyond 1.88$P_0$, a forward slip region is observed, which increases continuously.

The contact problem has some new features that have not been previously considered. The contact feature in the pinned supported beam is analogous to a steady rolling contact [21] in that the entire contact area comprises a stick and slip region. If we can determine the axial force in the two beam segments, i.e., between the contacts and between the contacts and supports, for each case and sum the tangential tractions in the contact, then the total tangential force determined should equal the difference between the axial force in the beam on each side of the contact. Fig. 6 shows two axial forces that are at the outside and inside edge of the contact area, respectively, and the total tangential tractions in the contact region; it shows that there will be an axial force everywhere with a discontinuity between the inside and outside edge of the contact region, which is equal to the total tangential force at the contact. The condition at the contact region primarily comprises rolling as the contact area moves while transmitting a tangential force. Thus, in the region that transitions to the contact region, which comprises the outside edge (leading edge) of the contact area, there exists a stick region, whereas at the trailing edge, contact is lost and slip occurs.

It is also interesting to compare the ratio of the tangential force to the product of the frictional coefficient and normal force ($T/F$) in the contact region with the ratio of the stick region area to the total contact region area ($A_{stick}/A_{total}$), as shown in Fig. 7. The curve of the ratio $A_{stick}/A_{total}$ is close to the curve of $1 − T/F$ when $P/P_0$ is greater than 3.0. The ratio is also compared to that of the stick region to the total.
3.2. Beam contact problem with a pinned support under cyclic loading

In order to investigate the effect of cyclic loading on the beam contact, the contact behaviors according to the load amplitude and mean cyclic load are studied. The cyclic load pattern is shown in Fig. 8. The mean load and load amplitude are defined as $P_{mean} = (P_{max} + P_{min})/2$ and $P_{amp} = (P_{max} - P_{min})/2$, respectively. The mean load can be selected from the three load regions specified for the monotonic loading in Section 3.1. When the mean load is in the range of $P/P_0 < 1.88$, the stick state is continuously maintained as the load evolves cyclically regardless of the load amplitude. If the mean load is in the range of $P/P_0 > 1.88$ and the load amplitude is small, the stick and slip region of the initial loading period changes to a stick region at all the points of the contact region within a few load cycles, as shown in Fig. 9. This figure shows the evolution of the contact states in the first and third cycles of loading and unloading, when the full load is $3.88P_0$ and load amplitude is $0.23P_0$. The maximum load $P_{max}$, which is the sum of the mean load and load amplitude, is $4.11P_0$. During the first loading, a stick region occurs at the leading edge and a forward slip region occurs at the trailing edge. When the unloading begins, the entire contact area changes into a stick region. In the next cycle of loading/unloading, the stick contact state does not change. During the full-stick cyclic state, the total tangential force decreases asymptotically until it reaches a limit. Fig. 10 shows the normalized contact pressure and contact shear distributions in the first cycle of loading and unloading when $P/P_0$ is 3.88, as shown in Fig. 9. The distribution of the normal contact traction is similar for both loading and unloading; however, the maximum contact pressure during unloading is lower than that during loading. Moreover, the maximum contact shear during unloading is lower than that during loading. If the load amplitude is large, the contact system asymptotically approaches a steady state with both stick and slip regions within a period.

If the load amplitude increases within the range of the mean load $P > 1.88P_0$, the contact state in the first cycle of loading is similar to that shown in Fig. 9. However, after several cycles, a steady state of contact is reached, in which a complex slip and stick state is observed. Fig. 11 shows the evolution of the contact states in the steady state of the loading/unloading phase for the mean load of $3.88P_0$ and load amplitude of (a) $0.54P_0$, (b) $0.66P_0$, and (c) $0.77P_0$. When the load amplitude is $0.54P_0$, a stick region is observed in the entire contact region during half of the loading cycle, and the forward slip region is observed from the middle of the contact region to the right stick region of the contact region. In the unloading phase, the entire contact region changes to the stick state and a small backward slip region occurs on the left side of the contact. When the load amplitude increases to $0.66P_0$ and $0.77P_0$, the slip regions for the load amplitude of $0.54P_0$ are extended. One peculiar phenomenon has been observed in which backward slip regions occur beyond the middle of the unloading. The contact states can be validated from the contact traction distribution. As an exemplary result of the contact traction distribution, Fig. 12 shows the normalized contact pressure and contact shear traction distribution in the steady state of (a) loading when $P/P_0$ is 4.50 and (b) unloading when $P/P_0$ is 3.26.
Fig. 11. Evolution of contact states in the steady state of loading/unloading phases for the mean load of 3.88$P_0$ and load amplitude of (a) 0.54$P_0$, (b) 0.66$P_0$, and (c) 0.77$P_0$ in the case of a pinned support; the contact states indicated include stick (1), separation (2), forward slip (3), and backward slip (4).

Fig. 12. Normalized contact pressure and contact shear traction distribution in the steady state of (a) loading when $P/P_0$ is 4.50 and (b) unloading when $P/P_0$ is 3.26.

Fig. 13. Axial forces $T_{\text{front}}$ and $T_{\text{rear}}$ at the outside and inside edge of the contact area, respectively, and total tangential tractions $\int q_0 d\xi$ in the contact region for the steady state of loading/unloading phases of the mean load 3.88$P_0$ and load amplitude 0.66$P_0$ in the case of a pinned support.

The transmitted tangential forces in the beam are investigated during the cyclic loading. Fig. 13 shows the difference between two axial forces in the beam and the total tangential tractions in the contact region during the steady state of the loading/unloading phase when the mean load and load amplitude are 3.88$P_0$ and 0.66$P_0$, respectively. In the loading and unloading phases, the differences between the two axial forces at the leading and trailing edges of the contact regions are negative or positive in specific ranges of $P/P_0$, thus causing a change in the
direction of the transmitted friction force during the cycle. Fig. 14 shows a set of distributions for the contact shear traction at several points in the loading and unloading cycles. When the frictional force is zero, there exist regions of positive and negative contact shear traction.

### 3.3. Effect of thickness on frictional energy dissipation in the pinned support case

This investigation of a beam contact problem shows the evolution of contact states that exhibit a smaller cyclic slip region in the steady state as compared to that in the initial cycle of loading. If the separation state is not observed in the contact region, this contact phenomenon can be explained by frictional shakedown [15]. Because these contact results are obtained by using the model of a beam of specific thickness, their dependence on the beam thickness is required to be identified. The effect of the beam thickness is represented by the frictional energy dissipation in one complete cycle of loading and unloading, $W_r$, which is defined as $-\int q(s)\mathrm{d}s$, where $q(t)$ is the slip velocity. The nondimensional frictional energy dissipation is defined as $W_r/((hP_{\text{amp}})^2/A_{\text{max}}E')$, where $E'$ is $E/(1-\nu^2)$, and $E$ and $\nu$ are the modulus of elasticity and Poisson's ratio, respectively. Both the maximum load and maximum contact areas are calculated for a steady state cycle. The nondimensional frictional energy dissipation for various $h/L$ is plotted logarithmically against the nondimensional load amplitude $P_{\text{amp}}/P_{\text{mean}}$ in Fig. 15. The most striking features of these results are that they all tend to straight lines with a positive slope of 5.7 and the frictional energy dissipation is the maximum at the thickness ratio of 0.021.

![Fig. 14. Distribution set for contact shear at several loads during the cycle of Fig. 13 for (a) loading and (b) unloading.](image)

![Fig. 15. Nondimensional frictional energy dissipation in the steady state cycle according to (a) nondimensional load amplitude and (b) nondimensional thickness at $P_{\text{amp}}/P_{\text{mean}}$ of 0.16.](image)

![Fig. 16. Evolution of contact area and contact states during monotonic loading in the roller support case.](image)

### 3.4. Beam contact problem with roller support and monotonic and cyclic loading

If the beam is supported by a roller, the contact behaviors observed are different from those of the pinned support model. In order to compare the results of the pinned support model with those of the roller support model, the variation in the contact states under monotonic loading is shown in Fig. 16, which shows the entire contact region as a stick state in all the ranges of loading. In addition, the contact region in the beam is closer to the support as compared to that in the case of the pinned support. This contact behavior, which comprises a stick state in
the previous sections, depending on the geometric specifications and the construction of the grid. Thus, it would be beneficial to the community of the field to examine practical examples in order to determine whether this idealized model provides useful insights in such cases. The current practical model has been adopted in a previous patent model [22], and it incorporates the material properties and specific dimensions of [20]. The current model (shown in Fig. 19) has a curved-shaped beam structure with a thickness of 0.12 mm, an arc length of 6 mm, and a roller support. The modulus of elasticity and Poisson’s ratio are 200 GPa and 0.3, respectively. The frictional coefficient between the rigid cylinder and the beam is 0.2, as already stated in Section 2. The load P is monotonically applied to the center of the cylinder.

The evolution of the contact states under monotonic loading for a roller-supported beam is shown in Fig. 20(a). When the load reaches 15.6 N, the contact area separates into two contact regions, and the load then increases to 21.7 N. During the loading, the entire contact region remains in a stick state. This is similar to the result shown in Fig. 16. Furthermore, if the beam thickness is increased to a value greater than 0.3 mm, stick and slip contact behavior is observed in the contact area. It is conjectured that as the curved-shaped beam structure has a greater stiffness, particularly at the kinked region, the bending mode is affected by the kinked region and the thickness of the beam. It is confirmed that a GTRF system with a small grid beam thickness shows no frictional energy dissipation. For the sake of comparison, the contact state evolution for the roller support and pin support are shown in Fig. 20(a) and (b), respectively. The separation of the contact area into two contact regions starts at the load of 4.36 N. When the load reaches 4.36 N, both forward slip and stick zones are observed in the contact area. These contact states are maintained until the force increases to a value of 6.54 N. For this practical GTRF model, the stick state of contact is observed during the loading when a roller support is considered, and this contact state is maintained even at large load magnitudes.

4. Conclusions

The frictional contact problem of a rigid cylinder and beam with a pinned or roller support under monotonic and cyclic loading was numerically investigated in this study. In the case in which one end of the beam is pinned, the initial loading, which separates the contact area into two regions, generates a rolling contact that generates a slip contact at
the leading edge of the contact surface and a stick state at the trailing edge. In the case of this new rolling contact behavior, the ratio of the stick area to the total contact surface is 1 – T/FF. After the cyclic loading evolves, the contact area at which separation does not occur reaches a cyclic steady state owing to the influence of frictional shakedown. This pinned-support condition generates considerable frictional energy dissipation and induces a maximum dissipation at a certain beam thickness. However, frictional energy dissipation is not observed in the roller support condition regardless of the beam thickness because a stick state is maintained in the entire contact region for all ranges of loading and unloading. In order to minimize fuel leakage in the case of grid-to-rod fretting, the use of a spacer grid cell with a roller support is recommended.

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References