

Power and Centrality: A Family of Measures¹

Phillip Bonacich
University of California, Los Angeles

Although network centrality is generally assumed to produce power, recent research shows that this is not the case in exchange networks. This paper proposes a generalization of the concept of centrality that accounts for both the usual positive relationship between power and centrality and Cook et al.'s recent exceptional results.

Cook et al. (1983) have shown that power does not equal centrality in exchange networks. In a set of experimental and simulation studies, those who were the most central were not the most successful in exercising bargaining power. This seems to contradict much social network research, especially in the area of interlocking directorates (Mizruchi 1982; Mintz and Schwartz 1985), that assumes that centrality is equivalent to power. Moreover, there is an extensive social psychological literature showing that, in experimentally restricted communication networks, the leadership role typically devolves upon the individual in the most central position (Leavitt 1951; Berkowitz 1956; Shaw 1964).

I propose a family of centrality measures $c(\alpha, \beta)$ generated by two parameters, α and β . The parameter β reflects the degree to which an individual's status is a function of the statuses of those to whom he or she is connected. If β is positive, $c(\alpha, \beta)$ is a conventional centrality measure in which each unit's status is a positive function of the statuses of those with which it is in contact.² In a communication network, for example, a

¹ Requests for reprints should be sent to Phillip Bonacich, Department of Sociology, University of California, Los Angeles, California 90024.

² In an influential paper, Freeman (1979) identified three aspects of centrality: betweenness, nearness, and degree. Perhaps because they are designed to apply to networks in which relations are binary valued (they exist or they do not), these types of centrality have not been used in interlocking directorate research, which has almost exclusively used formula (2) below to compute centrality. Conceptually, this measure, of which $c(\alpha, \beta)$ is a generalization, is closest to being a nearness measure when β is positive. In any case, there is no discrepancy between the measures for the four networks whose analysis forms the heart of this paper. The rank orderings by the

positive value of β is appropriate because the amount of information available to a unit in the network is positively related to the amount of information available to those with which it has contact. In a power hierarchy, one's power is a positive function of the powers of those one has power over. Whenever one's centrality or power is increased positively by connections to high-status others, a positive value of β is called for.

However—and this is the major innovation in this paper—in bargaining situations, it is advantageous to be connected to those who have few options; power comes from being connected to those who are powerless. Being connected to powerful others who have many potential trading partners reduces one's bargaining power.³ In these types of situations, a negative value for β is appropriate; each unit's status is reduced by the higher status of those to which it is connected.

The sign of β corresponds exactly to the distinction that Cook et al. (1983; p. 277) make between positive and negative exchange systems. To modify their definition slightly to apply to whole systems: A set of exchange relations is positive if exchange in one relation is contingent on exchange in others and negative if exchange in one relation precludes exchange in others. In communication networks, exchanged information is usually received from others, and so the system is positive, but, when exchanging a commodity with one person precludes exchange with another, the relation is negative. These would be modeled with positive and negative values of β , respectively.

The magnitude of β affects the degree to which distant ties are taken into account. If $\beta = 0$, $c_i(\alpha, \beta)$ is simply proportional to the degree of unit i , the number of others with which it is connected, regardless of their centralities. As β increases in magnitude, the centralities of these others are taken more into account, so that $c_i(\alpha, \beta)$ becomes a function of the indirect as well as the direct ties connecting it to the system. The magnitude of the parameter β reflects the degree to which $c(\alpha, \beta)$ is a local or global measure of status. If β is zero, then only the quality of one's direct ties to others matters, and the greater β , the greater the effect of the whole pattern within which one is embedded. This will be discussed in greater detail later in the paper.

The most important limitation is that the measure concerns itself only

nearness and betweenness criteria and by $c(\alpha, \beta)$ for moderately positive values of β are identical, and $c(\alpha, \beta)$ correlates perfectly with degree when $\beta = 0$.

³ The assumption that power can be reduced rather than increased through a connection to powerful others appears in both Caplow's and Gamson's well-known theories of coalition formation (Gamson 1969; Caplow 1968). Because each actor wishes to play as dominant a role in his coalition as possible, powerful actors tend to be avoided as coalition partners.

with *network*-derived importance and ignores all other aspects that can affect the centrality or power of units in a positively or negatively connected network. For example, in positively connected communication networks, $c(\alpha, \beta)$ will not reflect communication ties to those outside the system or differences in the quality of information provided. In an organizational power system, $c(\alpha, \beta)$ will not reflect differences in rights and duties. In a negatively connected exchange network, $c(\alpha, \beta)$ will not be affected by the differing values of the goods individuals are offering. For these reasons, $c(\alpha, \beta)$ may give a very misleading picture of the pattern of centrality or power in a system unless network members in it are equal in these other relevant factors.

THE MEASURE

I have proposed (Bonacich 1972*a*, 1972*b*) a measure of centrality (in this paper, I will call it “ e ”) in which a unit’s centrality is its summed connections to others, weighted by their centralities. It has become the standard measure of centrality in interlocking directorate research (Mintz and Schwartz 1985, p. 263). Let R be a matrix of relationships. R is usually but not necessarily symmetric. The main diagonal elements of R are zeros. The centrality of unit i is given by the following expression:

$$\lambda e_i = \sum_j R_{ij} e_j, \quad (1)$$

where λ is a constant required so that the equations have a nonzero solution. In matrix notation,

$$\lambda e = Re, \quad (2)$$

where e is an eigenvector of R , and λ is its associated eigenvalue. The largest eigenvalue is usually the preferred one.⁴

The measure I am proposing allows more flexibility. A parameter β allows one to vary the degree and direction (positive or negative) of the dependence of each unit’s score on the score of other units.⁵

⁴ All eigenvectors of R give solutions consistent with eqq. (1) and (2). They are all possible centrality measures. However, if R is symmetric, each eigenvector is a factor of R , and the associated eigenvalue measures the accuracy with which it can reproduce R .

⁵ The distinction between centralities when β is either positive or negative is not the same as Knoke and Burt’s distinction between centrality and prestige as two different types of prominence (Knoke and Burt 1983). For Knoke and Burt, centrality is measured within a system of symmetric relations, whereas high prestige is acquired by receiving unreciprocated choices from others. The measure $c(\alpha, \beta)$ is appropriate for

$$c_i(\alpha, \beta) = \sum_j (\alpha + \beta c_j) R_{ij}. \quad (3)$$

In matrix notation,

$$c(\alpha, \beta) = \alpha(I - \beta R)^{-1} R \mathbf{1}, \quad (4)$$

where “1” is a column vector of ones and I is an identity matrix.⁶

As can be seen from formula (4), the parameter α affects only the length of the vector $c(\alpha, \beta)$. In the following analyses, α is selected so that

$$\sum_i c_i(\alpha, \beta)^2,$$

the squared length of $c(\alpha, \beta)$, equals the number of units in the network. Therefore, $c_i(\alpha, \beta) = 1$ means (approximately) that position i does not have an unusually large or small degree of centrality, irrespective of the number of positions in the network.

MATHEMATICAL INTERPRETATIONS OF $c(\alpha, \beta)$

Under some conditions, the parameter β can be interpreted as a probability and $c(\alpha, \beta)$ as the expected number of paths in a network activated directly or indirectly by each individual. This is possible because $c(\alpha, \beta)$ is an infinite sum when β is less in absolute value than the reciprocal of the largest eigenvalue of R . Thus,

$$c(\alpha, \beta) = \alpha \sum_{k=0}^{\infty} \beta^k R^{k+1} \mathbf{1} = \alpha (R \mathbf{1} + \beta R^2 \mathbf{1} + \beta^2 R^3 \mathbf{1} + \dots). \quad (5)$$

The total number of direct and indirect paths from position i is $c_i(\alpha, \beta)$ when each path is weighted inversely to its length.⁷ Thus, $c(\alpha, \beta)$ is a

both symmetric and asymmetric relations. If there are asymmetric relations, as in fig. 1, $c(\alpha, \beta)$ with $\beta > 0$ will measure prestige, and, if relations are symmetric, it will measure centrality.

⁶ The vector $c(\alpha, \beta)$ approaches e as a limit as β approaches the reciprocal of the largest eigenvalue of R .

⁷ A path may pass through the same point more than once. The quantity $c(\alpha, \beta)$ is related to Katz's (1953) measure of status t . Thus,

$$\begin{aligned} t &= \sum_{i=1}^{\infty} \beta^i R^i \mathbf{1} \\ &= \beta^{-1} c(\mathbf{1}, \beta), \end{aligned}$$

where β is less in absolute value than the reciprocal of the largest eigenvalue of R . When β is positive, t and $c(\alpha, \beta)$ are perfectly correlated, and, when β is negative, they

closeness measure of centrality in Freeman's (1979) sense; it is large when the paths connecting it to other positions are the highly weighted short paths.

When $\beta > 0$, $c(1, \beta)$ and β have simple expected value interpretations. Assume that individuals in the network R communicate with all those with whom they are connected and that β is the probability that a communication, once sent, will be transmitted by any receiving individual to any of his contacts. A communication network is positively connected; a message cannot be sent until it is received. R_1 is the number of direct paths initiated by each individual. The quantity βR^2_1 is the expected number of these communications that are passed on to others. The expected number of messages transmitted at the k th remove is $\beta^{k-1}R^k_1$. Therefore, the total number of communications caused by each individual is given by the following vector:

$$\sum_{k=1}^{\infty} \beta^{k-1}R^k_1 = \sum_{k=0}^{\infty} \beta^k R^{k+1}_1 = c(1, \beta), \quad (6)$$

where $c(1, \beta)$ is simply the total number of communications in the whole network directly or indirectly caused by each individual if β is the probability that a communication is transmitted. In an asymmetric power structure (another type of positively connected network), where β is the probability that a command will be successfully transmitted to subordinates, $c(1, \beta)$ is the total number of successful direct and indirect influences produced by each individual.

The magnitude of β should reflect the degree to which authority or communication is transmitted locally or to the structure as a whole. Small values of β heavily weight the local structure, whereas large values take more into account the position of individuals in the structure as a whole. β can be thought of as a radius within which the researcher wishes to assess centrality. The expected length of any single path emanating from positions in the network is $(1 - \beta)^{-1}$. In this sense, β , or, more precisely, $(1 - \beta)^{-1}$, can be thought of as a radius within which power or centrality is being assessed. If $\beta = 0$, only direct connections are used to assess centrality; larger values correspond to larger radii of concern.

For example, in a communication network, a low positive value of β would be appropriate if most communication was local and not transmitted beyond the dyad. Larger values of β would be appropriate if com-

are perfectly negatively correlated. For Katz, β was merely a convenient "attenuation factor" that permitted the infinite sum to converge. He gave it no interpretation. He did not realize that it could take negative values or that variations in β would affect the way S ordered the members of a network.

munication traveled longer distances. When asymmetric power relations are being studied, the value of β should be a function of the transitivity of power relations. In an informal structure in which power is a characteristic only of dyads, a value near zero would be suggested, whereas in a formal hierarchy, in which the power of those one has power over matters because orders are likely to be transmitted, a larger value of β would be appropriate.

The following example may help clarify these issues. It is an asymmetric hierarchy of three levels, in which *A* has one subordinate, *B*, two subordinates, and *C* and *D*, no subordinates.

In dismissing Katz's (1953) measure of status, which is closely related to $c(\alpha, \beta)$ when $\beta > 0$, Taylor (1969) wrote that "Katz's index would probably attribute higher status to the head of the secretarial pool than to the president of the organization." However, this is true only when status is measured by the number of subordinates ($\beta = 0$). To the extent that status is a function of the status of one's subordinates, $c(\alpha, \beta)$ will reflect the level in a hierarchy as well as the number of subordinates. Table 1 gives values of $c(\alpha, \beta)$ for the structure in figure 1. For small values of β , position *B*, with the largest number of subordinates, is the most central. At $\beta = .50$, *A* becomes more central and increases its lead over *B* as β increases.

These interpretations of β as a probability measure of transitivity and of $c(\alpha, \beta)$ as the expected number of communications or influences directly and indirectly caused by an individual are not valid when β is negative. It follows from equation (4) that

$$c(\alpha, \beta) = \alpha R1 + \beta R c(\alpha, \beta) \tag{7a}$$

$$= \alpha R1 + \alpha \beta R^2 1 + \beta^2 R^2 c(\alpha, \beta). \tag{7b}$$

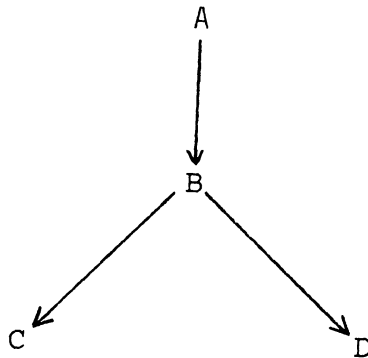


FIG. 1.—An asymmetric power hierarchy with three levels

TABLE 1
CENTRALITY SCORES FOR FIGURE 1 FOR VARIOUS
VALUES OF β

β	POSITION			
	A	B	C	D
089	1.79	.00	.00
.2	1.15	1.64	.00	.00
.4	1.34	1.49	.00	.00
.6	1.48	1.35	.00	.00
.8	1.59	1.22	.00	.00

When $\beta < 0$, even powers of R are weighted negatively and odd powers positively (5). Thus, having many direct ties contributes to centrality (power), but, if one's connections themselves have many connections, so that there are many paths of length two, centrality is reduced. When $\beta < 0$, $c(\alpha, \beta)$ is reduced when the connections of any unit are themselves central (7a) but increased by the centrality of those at distance two (7b), whose centrality has reduced the centrality of those at distance one. Substantively, one can be powerful in a bargaining network because those one is in contact with have no options or because their other optional trading partners themselves also have many other options.

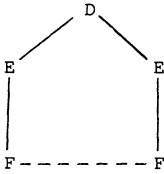
THE COOK ET AL. DATA: A NEGATIVE β

In their paper on power and centrality, Cook et al. point to the contrasting advantages of centrality and power-dependence concepts in understanding power: "The difficulty with power-dependence concepts, as they now stand . . . is that they are too closely bound to dyadic analysis. . . . In contrast, the approach to power through point centrality of positions has the virtue of taking the structure of the entire network into account in specifying at once a degree of centrality (and thus a power level) for every position in that structure. Because of the formal mathematical properties of networks, such analysis can be applied to very complex structures" (1983, p. 289). Later in the paper, they call for "a more general conception of centrality" (p. 298). That is precisely what this article attempts to supply.

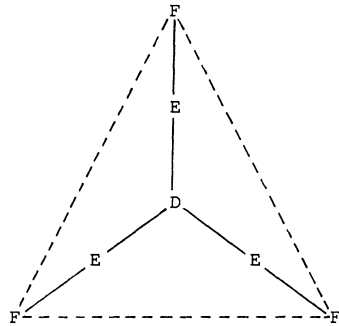
Four networks used by Cook et al. are shown in figure 2.⁸ A line between two positions means that they could engage in a transaction to

⁸ Two are omitted (1a and 1b) because they are uninteresting; in network 1a, one central person is connected to the other three and is most central under any definition of centrality, and in 1b all pairs are connected.

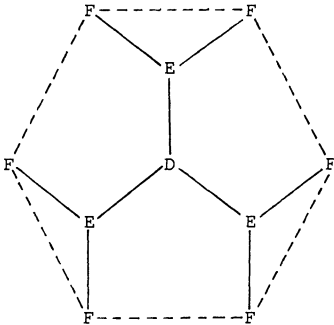
1(c) 5 person network
(three positions)



1(d) 7 person network
(three positions)



1(e) 10 person network
(three positions)



1(f) 13 person network
(three positions)

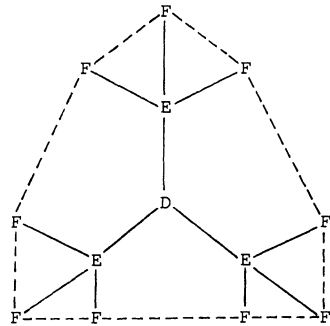


FIG. 2.—Exchange networks studies by Cook et al. 1983

divide a constant number of points. Each position could engage in only one negotiation per trial. A solid or broken line represents the opportunity to divide 24 and eight points, respectively. As in the Cook et al. paper, only the more profitable solid lines are used to compute centrality scores.

In all four of these networks, the rank order of positions by all conventional measures of centrality is $D > E > F$.⁹ If centrality were to correspond to bargaining power, this should also be the ordering for gains in

⁹ This is the order according to the nearness and closeness criteria and for the measures e and $c(\alpha, \beta)$ when β is moderately positive.

the series of bargaining trials. Table 2 gives the mean profit for those in position *E* per exchange with those in positions *D* and *F*, based on experiments using network 1*c* and simulations using all four networks (tables 1 and 2 in Cook et al. [1983]). In contradiction to predictions based on centrality, position *E* realized the most profit in all four networks, not position *D*, the most central.

Table 3 gives centrality scores $c(\alpha, \beta)$ for values of β with absolute values less than the reciprocal of the largest eigenvalue of R .¹⁰ In these calculations, $r_{ij} = 1$ if a relationship between positions i and j is present, $r_{ij} = 0$ if there is no relationship between positions i and j , and $r_{ii} = 0$ for all main diagonal elements of R .

Network 1*c*

Network 1*c* demonstrates the utility of allowing negative values for β . When $\beta = 0$, the *D* position and the two *E* positions are equally central because $c(\alpha, \beta)$ is proportional to degree. For $\beta > 0$, $c(\alpha, \beta)$ is a conventional centrality measure; *D* is more central than *E*, which is, in turn, more central than *F*. For $\beta < 0$, however, *E* is more "central" than *D*, and, as β decreases, the difference in centrality between *E* and *D* increases. The *F* positions are noncentral (powerless) because each has only one connection to others and hence has no bargaining alternatives. The *E* positions are powerful as a consequence of the powerlessness of the *F* positions to which they are connected. Finally, the *D* position is relatively powerless because it is connected to the two powerful *E*'s. The quantity $c(\alpha, \beta)$ for $\beta < 0$ predicts the Cook et al. experimental results, as can be seen by comparing tables 2 and 3; position *E* is both more central for negative values of β and more profitable.

Network 1*d*

In network 1*d*, $c(\alpha, \beta)$ is the same for all values of β . Formula (5) shows that, when R^21 is proportional to $R1$, as in network 1*d*, $c(\alpha, \beta)$ is proportional to $R1$, a vector of the degrees of the points, regardless of β . In improving on degree as a measure of centrality, $c(\alpha, \beta)$ makes use of differences between $R1$ and R^k1 . When there are no differences, $c(\alpha, \beta)$ will not be an improvement.¹¹

¹⁰ I will maintain this limit on β throughout the paper. Although not strictly necessary in formula (3), without it the infinite series in (5) does not converge (Golub and Van Loan 1983, p. 390), and so $c(\alpha, \beta)$ loses some of its interpretations.

¹¹ The same is true for the eigenvector measure of centrality e in eq. (2); if R^21 and $R1$ are proportional, e is proportional to $R1$, so that weighting ties by centralities does not produce an improved measure of centrality.

TABLE 2

MEAN PROFIT OF PERSON *E* PER EXCHANGE WITH *D* AND *F*: EXPERIMENTAL RESULTS FOR NETWORK 1*c* AND SIMULATION RESULTS FOR 1*c*, 1*d*, 1*e*, AND 1*f* FROM COOK ET AL. 1983

POSITION	NETWORK				
	Experimental	Simulation			
	1 <i>c</i>	1 <i>c</i>	1 <i>d</i>	1 <i>e</i>	1 <i>f</i>
<i>D</i>	13.97	16.96	15.21	18.60	19.34
<i>F</i>	15.43	17.94	16.97	19.22	20.00

I suggest that 1*d* be modified so that the *F* positions contribute to the centralities of the *E* positions, but not vice versa. Although an ad hoc solution, this change to an asymmetric *R* does eliminate the undesirable condition (the proportionality of $R1$ and R^21) without doing violence to the network; it forces the peripheral *F* positions to have zero centralities.

With this small and reasonable modification, $c(\alpha, \beta)$ behaves in the desired manner. The *F* positions, with no alternative trading partners, have zero centrality. Most important, for sufficiently negative values of β ($\beta = -.4$ and $\beta = -.5$ in table 4), the *E* positions are more central than the *D* position, which corresponds to the empirical results in table 2.

Network 1*e*

The results for network 1*e* are similar to those for 1*c*. When $\beta = 0$, the centralities of positions *D* and *E* are equal. When $\beta > 0$, $c(\alpha, \beta)$ is a conventional centrality measure, and position *D* is more central than the positions of type *E*. When, however, $\beta < 0$, position *D* is less central than the *E* positions because the *E* positions are connected to the powerless *F* positions. This is consistent with the data in table 2.

Network 1*f*

With respect to network 1*f*, $c(\alpha, \beta)$ is a conventional centrality measure for all values of β greater than .2; *D*, even though it has smaller degree than the *E*'s, is more central. For values of .2 or less, however, the *E* points are more central, and this greater centrality corresponds to the greater profits for these positions reported by Cook et al. in table 2.

In one way, $c(\alpha, \beta)$ does a better job of predicting the empirical results than do Cook et al.'s own hypotheses. They hypothesize that those in positions *D* and *F* are equally powerful in relation to the more powerful *E*

TABLE 4
CENTRALITY SCORES FOR MODIFIED NETWORK 1*d*

β	POSITION		
	<i>D</i>	<i>E</i>	<i>F</i>
-.500	1.53	.00
-.4	1.05	1.40	.00
-.3	1.41	1.29	.00
-.2	1.58	1.23	.00
-.1	1.67	1.84	.00
0	1.73	1.16	.00
.1	1.77	1.13	.00
.2	1.80	1.12	.00
.3	1.83	1.10	.00
.4	1.85	1.09	.00
.5	1.86	1.09	.00

(Cook et al., p. 285). Yet, in the experiment and in all four simulations, *E* gains less in his exchanges with *D* than in his exchanges with *F*, which indicates that *D* has more exchange power than *F*. This unexplained consistency in the findings is compatible with the ordering of the positions by centrality in tables 3 and 4 when β takes sufficiently large negative values.

CONCLUSIONS

To some, the measure $c(\alpha, \beta)$ may seem hopelessly ambiguous; $c(\alpha, \beta)$ can give radically different rankings on centrality, depending on the value of β . However, the measure accentuates an inherent ambiguity in the concept of centrality. There are different types of centrality, depending on the degrees to which local and global structures should be weighted in a particular study and whether that weight should be positive or negative. When communication is typically over long distances, position in the global structure should count more than when all communication is local. In an organized hierarchy in which power is transitive, the power of those one has power over should be weighted more highly in determining overall power than when all relations are dyadic. Finally, there will be situations in which power is increased by association with powerful others and situations in which it is decreased. There is no point in subsuming all these situations under one measure. Yet, it is also true that there is a core similarity in all these situations: one's status is a function of the status of those one is connected to. It is this common meaning that $c(\alpha, \beta)$ attempts to capture.

Another approach would be to use the best-fitting values of β to characterize or compare different structures. Variations in the optimal value of β would correspond to differences in system integration. For example, Roy and Bonacich (1985) found that, in the late 19th and early 20th centuries, central railroads in the structure of interlocking directorates were not particularly important because power resided in separate and balkanized communities of interest; instead of being powerful, central firms were peripheral to all power centers. Powerful firms were at the centers of the separate communities of interest. Although Roy and Bonacich used a completely different measure of centrality, their results correspond to a situation in which negative rather than positive values of β predict power. If communities of interest declined in significance and the system became better integrated later in the 20th century, the optimal value for β becomes more positive.

REFERENCES

- Berkowitz, L. 1956. "Personality and Position." *Sociometry* 19:210–22.
- Bonacich, P. 1972a. "A Technique for Analyzing Overlapping Memberships." Pp. 176–85 in *Sociological Methodology*, edited by Herbert Costner. San Francisco: Jossey-Bass.
- . 1972b. "Factoring and Weighting Approaches to Status Scores and Clique Identification." *Journal of Mathematical Sociology* 2:113–20.
- Caplow, T. 1968. *Two Against One*. Englewood Cliffs, N.J.: Prentice-Hall.
- Cook, K. S., R. M. Emerson, M. R. Gilmore, and T. Yamagishi. 1983. "The Distribution of Power in Exchange Networks: Theory and Experimental Results." *American Journal of Sociology* 89:275–305.
- Freeman, L. C. 1979. "Centrality in Social Networks: Conceptual Clarification." *Social Networks* 1:215–39.
- Gamson, W. 1969. "A Theory of Coalition Formation." *American Sociological Review* 26:373–82.
- Golub, G. H., and C. F. Van Loan. 1983. *Matrix Computations*. Baltimore: Johns Hopkins University Press.
- Hubbell, C. H. 1966. "An Input-Output Approach to Clique Identification." *Sociometry* 28:377–99.
- Katz, L. 1953. "A New Status Index Derived from Sociometric Analysis." *Psychometrika* 18:39–43.
- Knoke, D., and R. S. Burt. 1983. "Prominence." Pp. 195–222 in *Applied Network Analysis*, edited by Ronald S. Burt and Michael J. Minor. Beverly Hills, Calif.: Sage.
- Leavitt, H. J. 1951. "Some Effects of Certain Communication Patterns on Group Performance." *Journal of Abnormal and Social Psychology* 46:38–50.
- Mintz, B., and M. Schwartz. 1985. *The Power Structure of American Business*. Chicago: University of Chicago Press.
- Mizruchi, M. 1982. *The American Corporate Network*. Beverly Hills, Calif.: Sage.
- Roy, W., and P. Bonacich. 1985. "Centrality and Power in a Balkanized Network: Interlocking Directorates among American Railroads, 1886–1905." Paper presented at the annual meeting of the American Sociological Association, Washington, D.C.
- Shaw, M. E. 1964. "Communication Networks." Pp. 111–47 in *Advances in Experimental Social Psychology*, vol. 1. Edited by L. Berkowitz. New York: Academic.
- Taylor, M. 1969. "Influence Structures." *Sociometry* 32:490–502.