Optimal Opportunistic Scheduling and Adaptive Modulation Policies in Wireless Ad-Hoc Networks with Network Coding*

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SUMMARY In this paper, we study an opportunistic scheduling and adaptive modulation scheme for a wireless network with an XOR network coding scheme, which results in a cross-layer problem for MAC and physical layers. A similar problem was studied in [2] which considered an idealized system with the Shannon capacity. They showed that it may not be optimal for a relay node to encode all possible native packets and there exists the optimal subset of native packets that depends on the channel condition at the receiver node of each native packet. In this paper, we consider a more realistic model than that of [2] with a practical modulation scheme such as M-PSK. We show that the optimal policy is to encode native as many native packets as possible in the network coding group into a coded packet regardless of the channel condition at the receiver node for each native packet, which is a different conclusion from that of [2]. However, we show that adaptive modulation, in which the constellation size of a coded packet is adjusted based on the channel condition of each receiver node, provides a higher throughput than fixed modulation, in which its constellation size is always fixed regardless of the channel condition at each receiver node.

key words: network coding, wireless ad-hoc networks, opportunistic scheduling, adaptive modulation

1. Introduction

Since the publication by Ahlswede et al. [3], network coding has emerged as a promising solution to achieve higher throughput in both wireline and wireless networks. In the traditional network without network coding, a relay node forwards packets that it received from previous hop nodes to the appropriate next hop nodes without any processing in the information part of packets. However, in the network with network coding, a relay node forwards coded packets produced by performing some processing and combining several packets that it has already received. Even though network coding requires coding and decoding processes additionally in each node in the network, it has been shown that if we use an appropriate protocol considering network coding, we can improve network performance significantly [3]–[6]. Especially, in wireless network, if we appropriately exploit the broadcasting nature of wireless transmissions with network coding, we can improve network throughput more significantly by reducing the number of required transmissions [4], [5].

Among various network coding schemes, XOR network coding is one of the simplest network coding schemes. Despite of its simplicity, if we use XOR network coding with appropriate protocols, we can improve the throughput of the wireless network significantly. In [6], COPE, which is a protocol between the IP and MAC layer with XOR network coding for wireless ad-hoc networks, is proposed. In this protocol, each node stores packets that it overheard even though they are not forwarded to it, which is called opportunistic listening, and periodically reports which packets it currently stores to its neighbor nodes. Hence, each node has information on packets that each of its neighbor nodes stores. By using this information, a relay node can encode several native packets into a single coded packet and broadcasts the coded packet to next hop nodes of native packets in the coded packet in a single transmission, which results in reducing the number of required transmissions. This procedure is called opportunistic coding.

For example, there are three data flows in Fig. 1. Node A wants to send packet P4 to node 4, node B wants to send packet P3 to node 3, node C wants to send packet P2 to node 2. There is only one relay node, node 1 that should forward those three packets to appropriate next hop nodes. When node A transmits packet P4 to node 1, nodes 2 and 3 overhear it and store them in their packet pools. In similar ways, node 2 overhears packets P3 and P4, node 3 overhears packets P2 and P4, node 4 overhears packets P2 and P3 when packets are transmitted to node 1 from each of previous hop nodes A, B, and C. Overheard packets in each node are stored in its packet pool. We consider this specific situation and focus on relay node 1 that has three packets each of which should be forwarded to each of its neighbor nodes 2, 3, and 4. In this case, if node 1 knows this information through

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reports from each of its neighbor nodes and has a transmission opportunity, there exist several coding options and it is important to select the optimal coding option. If node 1 performs bit-wise XOR with native packets $P_2$ and $P_3$ to make coded packet $P_2 + P_3$ and broadcasts it, nodes 2 and 3 can decode the coded packet by bit-wise XORing it with packets that they have, i.e., with packet $P_3$ at node 2 and with packet $P_2$ at node 3, and retrieve each of native packets that should be forwarded to each of them, i.e., packet $P_2$ for node 2 and packet $P_3$ for node 3. Hence, node 1 can send two native packets with one transmission. On the other hand, if node 1 performs XOR network coding with native packets $P_2$, $P_3$, and $P_4$ and broadcasts coded packet $P_2 + P_3 + P_4$, then each of nodes 2, 3, and 4 can decode the coded packet to retrieve the native packet forwarded to each of them. Hence, in this case, node 1 can send three packets with one transmission and this is the optimal coding option in the situation in Fig. 1.

In COPE, which does not consider transmission errors in the physical layer, we can easily see that the optimal coding option at a relay node is to include the maximum number of $N$ native packets in its output queue that satisfies the following [2]:

- Each of $N$ native packets in the relay node should be forwarded to each of $N$ different neighbor nodes of the relay node.
- All the $N$ neighbor nodes should be in transmission range of the relay node.
- Each neighbor node should have the other $N - 1$ native packets except the native packet that should be forwarded to itself.

We call the set of neighbor nodes of the relay node that satisfies the above conditions the network coding group. Thus, the optimal coding option of a relay node at the MAC layer is to encode all possible native packets each of which should be forwarded to each of nodes in the network coding group. We call this coding option the MAC-optimal coding option.

However, in wireless network, it is well known that the consideration of the wireless channel condition is very important to improve network performance. Recently, this issue in network coding was studied in [2] through opportunistic scheduling, in which a relay node selects native packets to be encoded together considering the channel condition at each receiver node to which each native packet should be forwarded. In [2], the capacity of each link is assumed to follow the Shannon capacity. In other words, it is assumed that a packet is transmitted without any error if the data rate of a link is less than or equal to its capacity and otherwise, it cannot be transmitted at all. With this idealized model, [2] showed that opportunistic scheduling, in which native packets only for a subset of neighbor nodes in the network coding group are dynamically selected for a coded packet considering the channel condition at each neighbor node, is the optimal policy, i.e., it showed that the MAC-optimal coding option may not be an optimal coding option, if we consider the channel condition at each neighbor node in the network coding group. For example, in Fig. 1, if the channel condition between nodes 1 and 4 is bad, then, node 1 may obtain a higher throughput by encoding only native packets $P_2$ and $P_3$ into a coded packet and send it to only nodes 2 and 3, which is different from the MAC-optimal coding option in COPE.

In this paper, we study opportunistic scheduling and adaptive modulation policies in the network coding group, which is a similar problem to that of [2]. However, in this paper, we study this problem with a more realistic model than [2], considering practical modulation schemes such as M-PSK and its error probability. With this model, in fact, we make the following conclusion that is different from the conclusion of [2]. To maximize the throughput of a transmission, we should include all native packets for all neighbor nodes in the network coding group. In other words, the MAC-optimal coding option is still optimal even though we consider the channel conditions at receiver nodes. However, we also show that adaptive modulation considering the the channel condition at each neighbor node in the network coding group, provides higher throughput than fixed modulation.

The rest of this paper is organized as follows. In Sect. 2, we present our opportunistic scheduling and adaptive modulation. We provide numerical results in Sect. 3 and finally conclude in Sect. 4.

2. Opportunistic Scheduling and Adaptive Modulation

In this paper, we focus on a specific relay node in the network (e.g., node 1 in Fig. 1) and a specific situation in which the tagged relay node has $N$ neighbor nodes in its network coding group (e.g., nodes 2, 3, and 4 in Fig. 1). Hence, the tagged relay node has $N$ native packets each of which should be forwarded to each of its $N$ neighbor nodes in the network coding group. In addition, each of $N$ neighbor nodes in the network coding group has all $N - 1$ native packets in its packet pool except a packet that should be forwarded to it. Considering the above specific situation at the tagged relay node, we will study the optimal opportunistic scheduling and adaptive modulation strategy at the relay node.

Without loss of generality, we assume that $N$ neighbor nodes in the network coding group are ordered in a decreasing order of the SNR of the channel from the tagged relay node. We also assume that the transmission power of the tagged relay node, $P_{tx}$, is fixed. Hence, the SNR at each neighbor node $i$ in the network coding group, $\gamma_i$, is obtained as

$$\gamma_i = P_{tx} \left( \frac{h_i}{\sigma_i} \right)^2,$$

(1)

where $h_i$ and $\sigma_i$ are complex channel gain and standard deviation of AWGN at neighbor node $i$, respectively. We assume that all nodes are static and the fading of a link between nodes is slow enough to be fixed during the transmission of a packet. Hence, the SNR at each neighbor node is fixed during the transmission of a packet. We also assume that
the length of a packet, $K$ and the duration of a symbol, $T$ are fixed. Then, when the SNR is $\gamma_i$ and the modulation constellation size is $M_i$, the expected throughput at neighboring node $i$ in the network coding group, which is defined as the number of successfully transmitted packets per second, is obtained as
\[
 r_i(M_i, \gamma_i) = \frac{1}{KT} (\log_2 M_i) q_p(M_i, \gamma_i),
\] (2)
where $q_p(M_i, \gamma_i)$ is the packet transmission success probability when SNR is $\gamma_i$ and the constellation size is $M_i$. When the constellation size is $M_i$, since a symbol includes $\log_2 M_i$ bits, a packet is consists of $\lceil K/\log_2 M_i \rceil$ symbols. Hence, the packet transmission success probability is obtained as
\[
 q_p(M_i, \gamma_i) = (1 - p_e(M_i, \gamma_i))^{\lceil K/\log_2 M_i \rceil},
\] (3)
where $p_e(M_i, \gamma_i)$ is the symbol error probability\(^1\). With M-PSK, the symbol error probability is given by [7]
\[
 p_e(M_i, \gamma_i) = 2Q\left(\sqrt{2\gamma_i \cdot \sin \frac{\pi}{M_i}}\right),
\] (4)
where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$. Hence, if a coded packet includes native packets only for a subset of neighbor nodes 1, 2, $\cdots$, $n$ in the network coding group, the expected throughput for the transmission of a coded packet is obtained as:
\[
 r^p(M, \bar{\gamma}_n) = \frac{1}{KT} (\log_2 M) \sum_{i=1}^n q_p(M, \gamma_i),
\] (5)
where $M$ is the constellation size of modulation of the coded packet and $\bar{\gamma}_n = (\gamma_1, \gamma_2, \cdots, \gamma_n)$.

To maximize the expected throughput, we should determine the optimal subset of neighbor nodes in the network coding group to which a coded packet is transmitted and the constellation size of modulation of the coded packet in (5). Given SNR vector $\bar{\gamma}$, we first study how to select the optimal subset of neighbor nodes in the network coding group to which the coded packet is forwarded from the relay node, i.e., the opportunistic scheduling policy.

**Proposition 1:** For any given $\bar{\gamma} = (\gamma_1, \gamma_2, \cdots, \gamma_n)$, the optimal scheduling policy for the tagged relay node is to select all neighbor nodes in the network coding group and to encode native packets each of which should be forwarded to each of all neighbor nodes in the network coding group.

**Proof:** First, note that the expected instantaneous throughput at neighboring node $i$, $r_i(M_i, \gamma_i)$, is nonnegative for any $M_i$ and $\gamma_i$. Let us define $M_{\text{opt}}(\bar{\gamma}_n)$ as the optimal constellation size when the coded packet includes native packets only for neighbor nodes 1, 2, $\cdots$, $n$. Then, the expected throughput for the transmission of the coded packet defined in (5) is obtained as
\[
 r^p(M_{\text{opt}}(\bar{\gamma}_n), \bar{\gamma}_n)
 = \frac{1}{KT} (\log_2 M_{\text{opt}}(\bar{\gamma}_n)) \sum_{i=1}^n q_p(M_{\text{opt}}(\bar{\gamma}_n), \gamma_i).
\]

Hence,
\[
 r^p(M_{\text{opt}}(\bar{\gamma}_n), \bar{\gamma}_n)
 \leq \frac{1}{KT} (\log_2 M_{\text{opt}}(\bar{\gamma}_n)) \sum_{i=1}^n q_p(M_{\text{opt}}(\bar{\gamma}_n), \gamma_i)
 + \frac{1}{KT} \log_2 M_{\text{opt}}(\bar{\gamma}_n) \cdot q_p(M_{\text{opt}}(\bar{\gamma}_n), \gamma_{n+1})
 = \frac{1}{KT} (\log_2 M_{\text{opt}}(\bar{\gamma}_n)) \sum_{i=1}^{n+1} q_p(M_{\text{opt}}(\bar{\gamma}_n), \gamma_i)
 = r^{p+1}(M_{\text{opt}}(\bar{\gamma}_n), \bar{\gamma}_{n+1})
 \leq r^{p+1}(M_{\text{opt}}(\bar{\gamma}_n), \bar{\gamma}_{n+1}).
\]

Since given $\bar{\gamma}$, $r^p(M_{\text{opt}}(\bar{\gamma}_n), \bar{\gamma}_n)$ is nondecreasing in $n$, the optimal scheduling policy shown above is proved.

Since for any given $\bar{\gamma}$, the optimal scheduling is selecting all neighbor nodes in the network coding group, we can obtain the optimal constellation size for given $\bar{\gamma}$ by solving
\[
 M_{\text{opt}}(\bar{\gamma}) = \arg \max_M r^p(M, \bar{\gamma}_N).
\] (6)

By substituting the error probability of the modulation scheme that is used in the system, (e.g., for M-PSK, by substituting (4)), into (3), the expected throughput for the transmission of the coded packet for each constellation size can be calculated.

We can calculate the expected instantaneous throughput for the transmission of a coded packet by using the joint pdf of SNR’s as
\[
 r_{av} = \int_{\gamma_1}^{\gamma_N} \cdots \int_{\gamma_N} r^p(M_{\text{opt}}(\bar{\gamma}), \bar{\gamma}) p(\bar{\gamma}) d\gamma_1 \cdots d\gamma_N,
\] (7)
where $p(\bar{\gamma})$ is the joint pdf of SNR’s at neighbor nodes in the network coding group. When all SNRs at neighbor nodes are independent of each other and Rayleigh distributed, the joint pdf of SNR’s is obtained as a product of each marginal pdf in [7]:
\[
 p(\bar{\gamma}) = \prod_{i=1}^{n+1} \left( \frac{1}{\gamma_{av,i}} \right)^{\gamma_{av,i}/\gamma_{av,i}}
\] (8)
where $\gamma_{av,i}$ is the average SNR at neighbor node $i$.

### 3. Numerical Results

In this section, we present simulation results for our opportunistic scheduling and adaptive modulation scheme. We focus on the tagged relay node in the network when it has $N$ neighbor nodes in its network coding group. We model wireless links with i.i.d. Rayleigh distributions with the same average SNR. We set the size of a packet, $K$ as

\(^1\)In this paper, we do not consider the error correcting code. However, the error correcting code can be easily accommodated with a simple modification of this equation and the appropriate error probability.
The expected throughput for theoretical and simulated values of adaptive modulation.

Fig. 3 The expected throughput for the transmission of a coded packet with a different number of native packets in the coded packet.

1500 bytes (i.e., $12 \times 10^3$ bits) and the duration of a symbol, $T$ as $1/K$ sec, (i.e., we set $KT = 1$). To investigate the expected throughput of our scheme, for each simulation scenario, we perform 1000 times of simulations with randomly varying wireless channel conditions and calculate its average throughput. We use M-PSK as a modulation scheme, in which possible constellation sizes are 4, 8, 16, 32, and 64.

First, in Fig. 2, we compare the analysis results in (7) with our simulation results considering two different scenarios i.e., $N = 3$ and $N = 5$, varying the average SNR at neighbor nodes. As shown in this figure, the analysis results in (7) provides the almost same performance as the simulation results.

In Fig. 3, We show the optimality of our scheduling policy. We set the number of neighbor nodes in the network coding group as ten, (i.e., $N = 10$). With varying the average SNR at neighbor nodes, we plot the expected throughput for the transmission of a coded packet with only $k$ native packets whose receiver nodes have higher SNR’s among ten neighbor nodes. For each case, we use the optimal constellation size. As shown in this figure, as $k$ increases, the expected throughput of the coded packet also increases, which validates the optimality of our scheduling policy. In this figure, the graph for $k = 1$ represents the performance of the system without network coding. Hence, this figure also shows the performance gain of forwarding with network coding over a simple forwarding without network coding.

We now compare the performance of our adaptive modulation scheme with the scheme with a fixed constellation size, in which its constellation size is fixed regardless of SNR’s at receiver nodes. In Fig. 4, we set the number of neighbor nodes in the network coding group as five (i.e., $N = 5$). As shown in this figure, our scheme always provides a higher expected throughput than the scheme with a fixed constellation size. This implies that to maximize the throughput for the transmission of a coded packet, it is important to consider the channel condition at each of receiver nodes for the coded packet.

4. Conclusion

In this paper, we studied the opportunistic scheduling and adaptive modulation problem for wireless networks with network coding. We showed that to maximize the throughput for the transmission of a coded packet, a relay node should encode all native packets each of which should be forwarded to each of all neighbor nodes in its network coding group regardless of channel conditions at neighbor nodes. In other words, static maximal scheduling without considering channel conditions is always optimal. However, we also showed that the constellation size for modulation should be adjusted based on channel conditions of neighbor nodes in the network coding group.

In this paper, we focus only on the maximization of the throughput of the coded transmission without considering fairness among neighbor nodes in the network coding group. Hence, our scheme could be unfair to some neighbor nodes, even though it provides the maximum throughput. Developing a policy that provides both efficiency and fairness will be our future research topic.

References