Frictionally excited thermo-elastoplastic instability

Seong-ho Ahn, Yong Hoon Jang *
School of Mechanical Engineering, Yonsei University, 134 Shinchon-dong, Seodaemun-gu, Seoul 120-749, Republic of Korea

A R T I C L E   I N F O
Article history:
Received 22 July 2009
Accepted 2 November 2009
Available online 10 November 2009
Keywords:
Thermo-elastoplastic instability
Hot spot
Brakes and clutches

A B S T R A C T
A transient finite element simulation is used to solve the two-dimensional contact problem involving thermo-elastoplastic instability (TEPI) in frictional sliding system. The existence of plastic deformation below the critical speed for thermoelastic instability is independent of the size of initial perturbation. For the simulation of the first/second partial contact, the amount of initial perturbation affects only the time interval of the first partial contact and the second partial contact is reached earlier, regardless of the initial perturbation. In addition, it shows that the locations of hot spots after cooling are changed.

1. Introduction

A small sinusoidal perturbation in the otherwise uniform contact pressure between two sliding bodies is unstable if the sliding speed exceeds a certain critical value depending upon the wavelength of the perturbation. This instability, which results from the intersection of frictional heat generation, thermoelastic distortion and elastic contact, is known as frictionally excited thermoelastic instability or TEI [1].

TEI has been proven to be responsible for creating the so called hot spots and hot judder in frictional clutches and brakes, a phenomenon constituting a common problem of great practical importance. The formation of such localized hot spots is indicative of high local stresses that can lead to material degradation and eventual failure.

To resolve this problem, some researches have been performed. Barber [1] first demonstrated the mechanism of TEI. Anderson and Knapp [2] observed the permanent material deterioration caused by hot spots in automotive brakes. Dinwidie and Lee [3] made a direct observation of the transient behaviors of hot spots using a high-speed infrared camera. Severe hot spotting is observed to induce low frequency vibration called judder in automotive disk brakes. Burton et al. [4] have shown that a small perturbation in the otherwise uniform contact pressure between two sliding half-planes is unstable if the sliding speed exceeds a certain critical value called critical speed [1].

Lee and Barber [5] developed a model of a disk having finite dimension in automotive disk brakes and demonstrated that the layer model predicts critical speeds of the order of those observed in automotive disk brakes. Zagrodzki et al. [6] developed a model with a disk and frictional layer having finite thickness to observe a non-monotonic transition to a steady state with the separation of contact region. Recent investigation to functionally graded material on TEI has been started by Jang and Ahn [8], who showed that the maximum critical speed can be obtained when non-homogeneity reaches a certain value, where the thin coating of functionally graded material on core steel material slides against the conventional frictional material.

Generally, the formation of hot spots during engagement of brake causes localized heat and high stresses, leading to material having plastic behavior. Thus, it is reasonable to perform the analysis to predict the formation of hot spots by considering the plastic behavior of material. In this study, we shall extend the TEI analysis to the TEPI analysis and investigate the transient thermo-elastoplastic behavior during instability.

2. The geometric model

The two-dimensional model shown in Fig. 1 represents the geometric model of the sliding contact system. The two external layers, \( \Omega_1 \) and \( \Omega_2 \), move in the plane with a relative sliding speed, \( V \), with respect to layer \( \Omega_3 \). Layers \( \Omega_1 \) and \( \Omega_2 \) are comprised of the same frictional material to be considered as elastic material, while layer \( \Omega_3 \) is comprised of steel which behaves as a thermo-elastoplastic material. The thickness of the layer, \( \Omega_1 \) and \( \Omega_2 \) are both \( h \). The thickness of the layer \( \Omega_3 \) is \( 2a \). The model is geometrically symmetric with respect to the axis, \( y = h + a \).

The thermo-elastoplastic problem is assumed to be cyclically symmetric in the sliding direction \( x \) with wavelength \( L = C/m \), where \( C \) represents the perimeter of the disk, and \( m \) is the wavenumber.
3. Material properties

The material properties of frictional layer and steel disk are shown in Table 1.

We use a conventional model of isotropic plasticity. Following a standard multiplicative elastic–plastic decomposition, we assume the yield function $F$ is

$$F = \sigma_e - \sigma - \sigma_y = (\frac{1}{2} \sigma : \sigma') - r - \sigma_y$$

(1)

where $\sigma_e$ is effective Mises stress; $\sigma_y$ the yield stress of the stationary plate; $\sigma'$ the deviatoric part of the stress tensor; $r$ isotropic hardening function, which is the expansion of yield surface during plastic deformation, expressed as

$$r = \int h_i \, d\varepsilon$$

(2)

![Geometric model of sliding system](image)

Fig. 1. Geometric model of sliding system.

where $h_i$ is the strain hardening coefficient; $d\varepsilon$ the effective plastic strain rate, defined as

$$d\varepsilon_{ij}^p = \frac{n_{ij} \cdot C_{ij} \varepsilon_{ij}^p}{n_{ij} \cdot C_{ij} n_{i} + h}$$

(3)

where $C_{ij}$ is the elastic stiffness matrix; $\varepsilon_{ij}^p$ total strain increment written in Voigt notation; $n_{ij}$ normal tensor to yield surface written in Voigt notation, expressed by

$$n_{ij} = \frac{\partial F}{\partial \sigma_{ij}} = \frac{3 \sigma_{ij}}{2 \sigma_e}$$

(4)

Yielding occurs when the von Mises effective stress exceeds the local yield strength, corresponding to the updated effective plastic strain and temperature rise (i.e., $F > 0$). In this study, the linear isotropic hardening is assumed, so that $h$ is constant.

Generally, steel disks used in clutches and brakes show substantial reduction of yield limit as temperature increases and at temperature occurring in hot spot area it can drop to a fraction of that at ambient temperature. In the current analysis, we assume that the yield limit of steel is not considered as temperature dependent. This may induce a limited feature of results.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material properties of disk and frictional material.</td>
</tr>
<tr>
<td>Properties</td>
</tr>
<tr>
<td>Elastic modulus (G N/m$^2$)</td>
</tr>
<tr>
<td>Thermal expansion ($\mu$ C$^{-1}$)</td>
</tr>
<tr>
<td>Thermal conductivity (W/m C)</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>Thermal diffusivity ($\mu$m/s$^2$)</td>
</tr>
<tr>
<td>Yield strength (MN/m$^2$)</td>
</tr>
<tr>
<td>Hardening coefficient (G N/m$^2$)</td>
</tr>
</tbody>
</table>
4. The thermoelastoplastic analysis

The thermo-elastoplastic problem involves thermo-mechanical coupling. It requires developing separate algorithms for the thermal and the elastoplastic analysis. The heat conduction equation for the moving frictional layer has a convective term due to the motion of the layer with respect to the reference system. In the following sections, the thermal and elastic analyses of the model are discussed in detail.

4.1. Thermal analysis

A reference system $Oxy$ is defined for the whole model which includes all the layers. The selection of reference system is based on the behavior of contact pressure perturbation. It has been reported that the numerical efficiency is provided when the reference system is fixed to the better conductor [6]. In the model shown in Fig. 1, the reference system is fixed to the layer $\Omega_2$, which is therefore stationary.

The heat conduction equation for layer $\Omega_1$ and $\Omega_2$ in the friction material components written with respect to the stationary frame fixed to the steel layer is described by the form

$$K_i \left( \frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2} \right) = \rho_i c_p \left( \frac{\partial T_i}{\partial t} + V \frac{\partial T_i}{\partial x} \right), \quad i = 1, 3 \quad (5)$$

where $c_p$, $\rho$, and $K$ denote specific heat, density and thermal conductivity, respectively. For layer $\Omega_2$, the heat conduction equation becomes

$$K_2 \left( \frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial y^2} \right) = \rho_2 c_p \frac{\partial T_2}{\partial t} \quad \text{in} \ \Omega_2 \quad (6)$$

where the material properties in $\Omega_2$ are constant. In this simulation, the heat generation caused by mechanical dissipation associated with plastic straining is assumed to be small because the evolution of plastic deformation is slow, causing that heat generated by the plastic deformation has time to dissipate.

The boundary conditions associated with the heat conduction equation are due to the contact between layers and surrounding environment. The contact between layers at $\Gamma_c$ ($y = y_i, i = 1, 2$), causes heat generation due to friction. This can be expressed as

$$q_i = f N p_i \quad (7)$$

where $f$ is the coefficient of friction and $p_i$ is the contact pressure at the interface between layers one and two and between layers two and three.

The interfacial boundary conditions on $y = y_i$ depend on the status of mechanical contact. The boundary conditions at the contact surface have two cases. When the surfaces are in contact, the heat is generated at the contact interface and is written as

$$K_i \frac{\partial T_i}{\partial y} \bigg|_{y = y_i} - K_{i+1} \frac{\partial T_{i+1}}{\partial y} \bigg|_{y = y_i} = q_i, \quad i = 1, 2 \quad (8)$$

and temperature is continuous at the contact interface such as

$$T_i \big|_{y = y_i} = T_{i+1} \big|_{y = y_i} = 0, \quad i = 1, 2 \quad (9)$$

When the contact surfaces are separated, its boundary condition is

$$\frac{\partial T_i}{\partial y} \bigg|_{y = y_i} = \frac{\partial T_{i+1}}{\partial y} \bigg|_{y = y_i} = 0, \quad i = 1, 2 \quad (10)$$

On the boundaries $x = 0$ and $x = L$, the surfaces are imposed by cyclic symmetry ($\Gamma_1 = \Gamma_2$)

$$T_i \big|_{x = 0} = T_i \big|_{x = L}, \quad i = 1, 2, 3 \quad (11)$$

For the upper and lower edges of the body, the convective heat transfer is shown as a form of

$$q_i = h(T_i - T_\infty) \quad (12)$$

where $h$ is the convective heat transfer coefficient and $T_\infty$ is the ambient temperature of the surroundings.

The heat conduction Eq. (3) shows convective terms. This term implies serious difficulties when the standard Galerkin finite element algorithm is used. Especially, when the mesh size exceeds a certain critical value such that the Peclet number ($P_e = Ve/k$) is greater than 2, where $e$ and $k$ are a characteristic element length and the thermal diffusivity, respectively, the standard Galerkin does not provide an adequate discretization to the convective terms and numerical oscillations appear in the solution. To overcome the problem of oscillations caused by the convective terms, a Petrov–Galerkin algorithm is introduced [6]. This new algorithm uses an upwinding approach that replaces the central difference scheme with a backward-difference scheme for both convective and diffusive terms to control the numerical diffusion. In this analysis, we use a commercial finite element package, ABAQUS [10], to perform the analysis.

4.2. Elastoplastic analysis

A standard elastic contact formulation is used which imposes continuity of normal displacement $u_n$ across the contact interfaces if the contact condition is satisfied.

The mechanical boundary conditions for the elastic analysis in the $x$ and $y$ directions can be summarized as the following:

At the vertical boundaries $x = 0$ and $x = L$ conditions similar to cyclic symmetry conditions occur, but with unconstrained mean thermal expansion in the $y$ direction, i.e.

$$(u_y)_i |_{x = 0} = (u_y)_i |_{x = L} = \text{constant}, \quad i = 1, 2, 3 \quad (13)$$

$$(u_y)_i |_{x = 0} = (u_y)_i |_{x = L}, \quad i = 1, 2, 3 \quad (14)$$

where $(u_y)_i$ denotes the displacement of $y$ direction in the $i$th layer.

At the horizontal external boundaries $y = 0$ and $y = y_1$, the conditions are

$$(u_y)_i |_{y = 0} = \text{constant} \quad (15)$$

$$(u_y)_i |_{y = y_1} = 0 \quad (16)$$

with the applied total axial force, $P_y$ in the $y$ direction, being controlled.

5. Analysis procedure

In the simulation procedure, the thermal and elastoplastic contact problems are solved sequentially in time. In the thermal analysis, the heat flux at time $t + n\Delta t$, where $t$ is the time at which the solution is known and $n = 1, \ldots, N$, is approximated by

$q_i^{n+1} = \frac{N t + n\Delta t}{4 N t + n\Delta t} q_i^n \quad (17)$

where $q_i^n$ is the contact pressure at time $t$. Then, after calculating $T^{n+1}$, the elastic contact problem at the time $t + N\Delta t$ is solved.

In the simulation, a sinusoidal perturbation is triggered by perturbing the contact pressure in the initial step of thermal analysis. This initial condition reflects only the general pattern of the eigenmode of the interest and the actual mode forms gradually during the initial stage of the simulation [11]. The
thermal and elastoplastic contact problem is also solved by the finite element method using Abaqus [10].

6. Conditions of finite element model

The numerical model in ABAQUS code, is performed using user-defined subroutine UMAT for elastoplastic analysis, GAPCON, DFLUX and UMASFL for heat transfer, and FRIC for the heat generation at the sliding interface. The quadrilateral linear elements are used in both thermal and elastoplastic models and the total number of nodes and elements are 2449 and 2250, respectively. The numerical model which performs the transient thermoelastoplastic analysis is selected in terms of mesh and time step, which was recommended by Refs. [6,7]. More specifically, the finite element mesh must be designed to be capable of reproducing the strong variation of temperature in the skin layer of friction material. In the good conductor, much smaller temperature gradients in the y-direction are expected. Consequently, much larger element size is used in the good conductor. The mesh in sliding direction x is uniform in each component. It is recommended by Zagrodzki et. al. [6] that the element aspect ratio in the skin layer be \( e_x/e_y \geq 1 \) and the wavelength of lowest mode is divided into 50 elements. For solving convection dominated problem, it is also recommended that the time step \( \Delta t \) satisfy the condition of \( \text{Courant number} = V \Delta t/e_x \).

7. Results

7.1. Transient behavior of contact pressure dependent on sliding speed

The transient behavior of contact pressure amplitude is investigated according to several sliding speed as shown in Fig. 2. The amplitude of the perturbed contact pressure is defined as the difference between maximum and minimum contact pressure.

At sliding speed under the critical speed for thermoelastic model, \( V_{cr} = 4 \text{ m/s} \), the contact pressure perturbation becomes stable, leading to zero amplitude of contact pressure, as expected in TEI process. However, the mean contact pressure is applied continuously and the corresponding heat generation is accumulated, resulting in the increase of stresses in the disk and later yielding occurs. The existence of plastic deformation below the critical speed for the thermoelastic model is independent of the size of initial perturbation. In particular, plastic deformation occurs even if there is no initial perturbation. If the sliding speed is greater than the critical speed for the thermoelastic instability, the amplitude of contact pressure gradually increases and becomes unstable; After yielding, the system leads to partial contact. For the final state of process, it is speculated that plastic deformation represents work hardening which is the strengthening of a material, so everything will be elastic. Basically, enough work hardening will have occurred to bring the final state inside the elastic limit, probably leaving the system in partial contact.

A typical result for the effective stress distribution and deformation at the sliding speed of 10 m/s is shown in Fig. 3. Comparing with the results of Fig. 2, plastic deformation at the contact region, where the stress is above the yield stress of 250 MPa, starts before 0.1 s and then after the plastic regions are gradually enlarged. After 0.16 s, separation occurs and this state is maintained during the process.

7.2. Implication of plastic deformation to brake

Experience with real vehicles suggests that once a TEI event has been recorded (meaning that the driver heard and felt the vibration during a long stop), the brake would be more liable to generate the same problem on future stops. This is most likely due to plastic deformation that occurred during the first event or the
martensite transformation after cooling on the hot spot area, which leaves the rotor with a significant perturbation in shape. Especially, the martensite phase is generated to the direction of thickness, extending over several hundred micrometers during cooling after the TEI event and the volume of the martensite phase is increased in the hot spot area [2]. From the previous results, it is then as if the initial perturbation is larger, so measurable (by the driver) vibrations are reached earlier in the process. Thus, it would be beneficial to quantify this effect. By using our simulation, we will make runs with initial perturbation after cooling, runs with initial perturbation taken from the end condition of a run involving plastic deformation.

Fig. 4 and Table 2 show the pressure perturbation as a function of time during the two events. As the initial perturbation decreases, the first partial contacts are delayed but the second partial contacts are reached earlier, approaching to a regular interval of time. Therefore, it is concluded that the amount of initial perturbation affect only the time interval of first partial contact and the second partial contact is reached earlier, regardless of the initial perturbation.

### 7.3. Formation of hot spots

The plates of a typical multidisk wet clutch after a period of normal service may have several hot spots. In the picture of hot spots, the dark areas usually correspond to regions in which high local temperatures have been experienced. Evidence of surface melting can be found in extreme cases. In addition, transfer of friction material components and the products of overheated transmission fluid may be involved. Recently, some patterns of hot spots after the service of clutch for a heavy duty vehicle are reported [12]. It shows several series of hot spots at different locations overlayed on each other after multiple engagements of the clutch. Especially, after the first hot spots are formed, followed by cooling, then the second hot spots are formed at different locations with more dark areas.

In the simulation of TEPI, a similar feature can be reproduced. Fig. 5 shows the effective plastic strains and deformation of disk plate during cooling after the formation of hot spots. This shows that yielding is developed continuously and the deformation shape of disk after cooling is changed in phase, leading to the formation of hot spot at different locations. Furthermore, the deformation of disk after cooling is applied to the second perturbation, which is likely to be a larger perturbation than the initial one, inducing severe hot spots in the disk.

### 8. Conclusion

The transient numerical simulations are performed to investigate TEPI process in the two dimensional frictional sliding problem. Under the speed below the thermoelastic critical speed, the contact perturbation becomes stable but the system undergoes plastic behavior. The existence of plastic deformation below the elastic critical speed is independent of the size of initial perturbation. We also investigate the effect of first/second perturbation of contact pressure, leading that the amount of initial perturbation affects only the time interval of first partial contact and the second partial contact is reached earlier, regardless of the initial perturbation. A proof of evidence for the change of hot spot location after cooling is presented.

### Table 2
Comparison of evolution time between first partial contact and second partial contact.

<table>
<thead>
<tr>
<th>Perturbation (kPa)</th>
<th>1st partial contact (s)</th>
<th>2nd partial contact (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.167</td>
<td>0.122</td>
</tr>
<tr>
<td>20</td>
<td>0.258</td>
<td>0.130</td>
</tr>
<tr>
<td>2</td>
<td>0.344</td>
<td>0.133</td>
</tr>
</tbody>
</table>

Fig. 4. Variation of contact pressure amplitude after cooling and reloading.

Fig. 5. Distribution of effective plastic strains and deformation during cooling. $t_{\text{total}}$ is the total time of cooling.
Acknowledgments

Authors are pleased to acknowledge helpful comment and support from Prof. J.R. Barber at the University of Michigan and Dr. P. Zagrodzki at Friction Holdings, LLC.

References