Numerical analysis of quasistatic frictional contact of an elastic block under combined normal and tangential cyclic loading

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ABSTRACT

Contact between solid bodies has become important in manufacturing in recent years as joining processes involving frictional contact such as friction stir welding and ultrasonic welding have shown significant promise in a number of applications, including the automotive industry. In this study, contact between an elastic block and a rigid surface under normal and/or tangential cyclic loading with mean normal loading is investigated using a commercial FEM package, ABAQUS. The slip/stick status and frictional energy dissipation per cycle are investigated for select combined loading conditions, as well as for different block heights, and cyclic loading frequencies. The results indicate that the combined cyclic loading has the potential to improve ultrasonic weld quality.

1. Introduction

An understanding of contact between solid bodies has become important in manufacturing in recent years, such as ultrasonic welding. Ultrasonic welding is known to be an environmentally-friendly and low-energy-consumption joining process, and it has been one of the major joining processes in the electronic industry for several decades [1]. Since early studies on ultrasonic welding [2,3], its application has expanded to rapid prototyping [4], automotive industry [5,6], and micro-electro-mechanical system [7,8].

The bonding induced by ultrasonic wave is a solid phase welding process which is primarily accomplished by softening one or both of the weldments with ultrasonic energy or heat [9]. The heat generated by scrubbing two contacting surfaces during the bonding process could play a significant role for the diffusion at the contacting surfaces. It is known that increasing ultrasonic power and bonding time generally enhances the diffusion process, for better intermetallic phase and stronger bonds [10].

Thus, a problem of particular interest is the effect of contact induced by the vibrating or repetitive loads in ultrasonic welding. Especially, since the periodic loading cycle in a combination of normal and tangential loads may induces more inevitable “microslip”, leading to more influence on frictional energy dissipation in the contact surfaces, this type of loading cycle should be investigated.

A series of research of this kind is restricted to a scope of contact mechanics, after a simple model of Cattaneo–Mindlin [11,12]. Several works were followed by Deresiewicz [13], who showed that the entire contact area would remain in a state of stick as long as the normal load is increasing and then, Ciavarella [14] and Jäger [15] extended the solution method of Catteneo to describe any frictional elastic contact problem. However, as the present authors are aware, no previous studies have remarked on the effect of the combined normal and tangential cyclic loading on frictional energy dissipation in elastic contact bodies, even though there have been numerous studies involving various contact geometries and loading conditions [16–20].

In this study, an elastic rectangular block on a rigid planar surface under combined normal and tangential cyclic loading with mean normal loading is investigated using ABAQUS. A two dimensional configuration with Coulomb friction is assumed. The impact of tangential cyclic loading and a combined normal and tangential loading is examined. The frictional behavior is evaluated in terms of the evolution of slip/stick/open state distribution during a cycle, and frictional energy dissipation per cycle for select loading conditions. The effect of the block height and different cyclic loading frequencies on frictional energy dissipation per cycle are also examined. Finally, the feasibility of applying combined loading to ultrasonic welding is discussed to suggest an enhanced ultrasonic welding process.

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Nomenclature

\( F_t \)  
\( h \)  
\( N_t \)  
\( P_0 \)  
\( P_s \)  
\( Q_t \)  
\( t_p \)  
\( t' \)  
\( u_s, u_n \)  
\( u_{y, \text{max}} \)

tangential reaction of an elastic block (MPa)
hight of an elastic block (mm)
normal reaction of an elastic block (MPa)
mean normal loading (MPa)
normal cyclic loading (MPa)
tangential cyclic loading (MPa)
2\(\pi/\omega\), period of cyclic loading (sec)
time shifted to zero at the beginning of a cycle of interest (s)
nodal slip displacement (mm)
nodal displacement in \( x \) and \( y \) directions, respectively (mm)
maximum nodal displacement in \( y \) direction at a given time (mm)

\( v_s \)  
\( v_{s, \text{max}} \)  
\( W_\mu \)  
\( w \)  
\( x_0 \)  
\( \Delta t \)  
\( \mu \)  
\( \phi \)  
\( \omega \)  
\( \omega_P \)  
\( \omega_Q \)

slip velocity (mm/s)
maximum slip velocity over a cycle (mm/s)
frictional energy dissipation per cycle (MJ)
half width of an elastic block (mm)
location of a node in \( x \) coordinate for an undeformed block (mm)
time step (s)
friction coefficient
phase offset of normal loading with respect to tangential loading (rad)
angular speed of cyclic loading (rad/s)
angular speed of normal cyclic loading (rad/s)
angular speed of tangential cyclic loading (rad/s)

2. Background

ABAQUS has been successfully used for numerical analyses of manufacturing processes involving frictional contact, such as drawing [21], forming [22], friction stir welding [23] and ultrasonic welding [24]. A contact condition is modeled in terms of “normal” and “tangential” behaviors in ABAQUS. They are described in Ref. [25], and briefly summarized below.

The normal behavior model defines the contact pressure \( p \) between two surfaces at a point as a function of the “overclosure” \( z \) of the surfaces, i.e. \( p = p(z) \). Among available models, the “direct hard contact” defines the relation as

\[
p = 0 \quad \text{for} \quad z < 0 \quad \text{(open)}, \quad \text{and}
\]

\[
z = 0 \quad \text{for} \quad p > 0 \quad \text{(closed)}.
\]

The contact constraint is enforced with a Lagrange multiplier representing the contact pressure in a mixed formulation. The virtual work contribution can be written in the linearized form as [25]

\[
d\delta W = \delta p \, dz + dp \delta z
\]

Coulomb friction model assumes that no relative motion occurs if the equivalent frictional stress \( \tau_{eq} = \sqrt{\tau_1^2 + \tau_2^2} \) is less than the critical stress \( \tau_{eq} = \mu \tau \), i.e. \( \tau_{eq} < \tau_0 \). Where \( \tau_1 \) and \( \tau_2 \) are frictional stresses in the two orthogonal directions on a contact surface, and \( \mu \) and \( p \) are the friction coefficient and the contact pressure, respectively. Slip can occur if \( \tau_{eq} > \tau_{crit} \). For isotropic friction, the direction of the slip and the frictional stress \( (\tau_j) \) coincide, i.e.

\[
\frac{\tau_j}{\tau_{eq}} = \frac{\dot{\gamma}_j}{\dot{\gamma}_{eq}}
\]

where \( \dot{\gamma}_j \) is the slip rate in direction \( j \), and \( \dot{\gamma}_{eq} = \sqrt{\dot{\gamma}_1^2 + \dot{\gamma}_2^2} \) is the magnitude of the slip velocity.

In “friction formulation with Lagrange multiplier” tangential behavior model, Lagrange multipliers are used to enforce exact sticking conditions. The rate of virtual work with a constraint term enforced with Lagrange multipliers \( q_j \) can be written for a contact surface \( S \) as [25]

\[
d\delta W^* = \int_S \left[ k_0 \delta\tau_{ij} \delta\dot{\gamma}_{ij} + \delta q_j + \delta q_i \delta\dot{\gamma}_{ij} + \tau_j \delta\dot{\gamma}_{ij} \right] dS
\]

for the stick condition, where \( k_0 \) and \( \gamma_j \) are a reference stiffness internally selected by ABAQUS and a tangential slip in direction \( j \), respectively, and [25]

\[
d\delta W^* = \int_S \left[ \frac{\tau_{eq}}{\Lambda_{ij}} (\delta\tau_{jk} - n_j n_k) \delta\dot{\gamma}_{ij} dS_{ijk} + \left( \mu + \frac{\partial \mu}{\partial p} \right) n_j \delta\dot{\gamma}_{ij} dp + \frac{\partial \mu}{\partial \tau_{eq}} n_j n_k \delta\dot{\gamma}_{ij} dS_{ijk} + \tau_j \delta\dot{\gamma}_{ij} \right] dS
\]

for the slip condition, where \( n_j \) and \( n_k \) are the normalized slip directions, \( \Delta t \) is the time step, and \( \dot{\gamma}_{eq} \) is the Kronecker delta. The slip/stick status of an element is updated in ABAQUS as follows: if an element is currently in the stick condition and satisfies \( \tau_{eq} < \tau_{crit} \), then it is updated to the slip condition. If an element is currently in the slip condition and satisfies \( \Delta t < \tau_{crit} \), then it is updated to the stick condition. More information is available in Ref. [25].

In this study, an elastic body on a rigid surface under a cyclic normal loading examined by Ahn and Barber [26] is selected to validate the friction contact model of ABAQUS. Then, it is used to examine the tangential cyclic and combined loading cases.

3. Analysis

The configuration used in the analysis is shown schematically in Fig. 1(a), in two dimensions. The width \( 2w \) and height \( h \) of the block are 40 and 10 mm, respectively. Thus, the width, \( w \), of the loading region on the top surface of the block is 20 mm. Young’s modulus and Poisson’s ratio of the block are selected as 200 GPa and 0.3, respectively. The friction coefficient \( \mu \) between the block and surface is 0.35. The material properties and friction coefficient are assumed to be constant. The block is subjected to normal loading \( P(t) \) and/or tangential loading \( Q(t) \) as shown in the figure, and these are defined as follows:

\[
P(t) = P_0 + P_1 \cdot \sin(\omega t - \phi)
\]

\[
Q(t) = Q_1 \cdot \sin(\omega t)
\]

where \( P_0 \) is a mean normal loading that is positive, \( P_1 \) and \( Q_1 \) are normal and tangential cyclic loadings, respectively. \( \omega \) is the angular speed which is set as \( 2\pi \) rad/s for most of the cases in this study, and \( \phi \) is a phase offset of the normal with respect to the tangential cyclic loading. Fig. 1(b) illustrates the mesh used in the study, which consists of four-node plane strain elements (CPE4). The element size is selected as \( \Delta x = \Delta y = 0.125 \) mm, following Ahn and Barber [26]. Thus, the mesh in the figure has a total of 25,600 elements and 26,001 nodes. The surface in Fig. 1(a) is modeled as a rigid surface with boundary conditions
$u_x = u_y = \theta_{xy} = 0$, where $u_x$ and $u_y$ are linear displacements in $x$ and $y$ directions, respectively, and $\theta_{xy}$ is angular displacement in the $xy$ plane. The normal behavior option for “surface interaction” of ABAQUS is selected as “direct hard contact”, Eq. (1), to avoid normal penetration, and the tangential behavior option is selected as “friction formulation with Lagrange multiplier”, Eq. (2), which minimizes tangential displacement of the block under no-slip condition.

The system in Fig. 1(a), with mass density of 7.8 g/cm$^3$ for the block, has a natural frequency of 64 kHz and 72 kHz for $\mu = 0$ and $\infty$, respectively. Since the frequency of interest in this study is around 20 kHz, which is a typical value for ultrasonic welding, quasi-static conditions are assumed for the block. For the time dependent loading, Eq. (3), the number of discretization per cycle is 100. Since the quasistatic assumption prohibits the block from any gross slip, the loading condition of Eq. (3) is limited by

$$|Q(t)| < \mu \cdot P(t)$$

In addition, $P(t)$ should always be positive to avoid separation of the block from the surface. Fig. 2(a) illustrates the limits for select $P_1$ and $P_0$. Among the conditions in the figure, $Q_1 = 200$ MPa satisfies the condition, while $Q_1 = 480$ MPa exceeds the limits, and thus cannot be considered in this study. $Q_1 = 340$ MPa in the figure shows an example close to the critical value for the condition, i.e. $|Q(t)| \approx \mu \cdot P(t)$. It can be seen from the figure that a critical $P_1$ or $Q_1$ should satisfy the relations: $\mu P(t) = Q(t)$ and $\mu \cdot \partial P(t)/\partial t = \partial Q(t)/\partial t$ for given $P_0$, $\omega$, $\phi$, and $Q_1$ or $P_1$. Fig. 2(b) shows the critical $P_1$ and $Q_1$ in normalized form that satisfy the static limit and the relations for a given $P_0$ and $\phi$. Once a loading condition which is within the static limit is determined, Eqs. (3a) and (3b) are applied to the block in a step-by-step manner, i.e. a normal load is applied first at a magnitude of $P(t=0)$. Then the normal and tangential loads as described by the equation are applied to the block.

Two major outcomes of this study are the frictional energy dissipation per cycle ($W_m$) and distribution of slip/stick status on the contact surface. The frictional energy dissipation per cycle is obtained from an ABAQUS output “ALLFD” [25], or $W_m(n)$, which is the frictional energy dissipation accumulated from the beginning to time $t$. The frictional energy dissipation per cycle of the $n$th cycle, $W_m^n$, is then obtained as

$$W_m^n = W_m^n(n \cdot t_p) - W_m^n((n-1) \cdot t_p) = \int_{-\omega}^{\omega} |F_x(x,t') - v_s(x,t')| \, dt' \, dx$$

where $t_p$ is the period of cyclic loading, i.e. $t_p = 2\pi/\omega$, $x_s$ is location on the contact surface, and $v_s$ is slip velocity of a node in contact with the rigid surface. Since frictional energy dissipation per cycle converges as a simulation proceeds [26], simulations are performed over several cycles, mostly 10, in this study. The frictional energy dissipation per cycle $W_m$ is then obtained from the last cycle of the simulation. Another major output, slip/stick status on the contact surface, is determined by displacement and slip velocity, as summarized in Table 1. The slip velocity $v_s$ in the table is calculated from an ABAQUS output “CSLIP” [25], or slip displacement $u_s$ as follows:

$$v_s = \frac{u_s^{t} - u_s^{t-\Delta t}}{\Delta t}$$

where $u_s^t$ and $u_s^{t-\Delta t}$ are the slip displacements at time $t$ and $t-\Delta t$, respectively.

For convenience in showing multiple cycles, a shifted time $t'$ is defined as follows:

$$t' = t - (n_i - n_d) \cdot t_p$$

![Fig. 1.](image1) An elastic block on a frictional rigid surface and (b) corresponding ABAQUS mesh, with a zoomed-in detail view in the upper right corner.

![Fig. 2.](image2) Loading histories with the static limit for $\phi = \pi/2$. (b) Normalized normal and tangential cyclic loading at the static limit.
where \( n_i \) and \( n_d \) are the total number of cycles of a simulation, and the number of cycles to be shown in the result which is counted from the last cycle, respectively.

**4. Results and discussion**

**4.1. Normal or tangential cyclic loading**

In this section, normal or tangential cyclic loading is applied to the elastic block, together with a non-zero clamping load, i.e. \( Q_{i}=0 \) or \( P_{i}=0 \) with \( P_{0}>0 \), Eq. (3). The angular speed \( \omega \) of the equation is set as \( 2\pi \text{ rad/s} \) (1 Hz). The phase offset \( \phi \) is set as zero for tangential cyclic loading, and \( \pi/2 \) for normal cyclic loading, i.e. Eq. (3) can be rewritten as

\[
P(t) = P_{0} - P_{1} \cdot \cos(\omega t)
\]

(8)

for normal cyclic loading. Thus the normal loading increases during the first half of a cycle, and decreases during the second half. In other words, the first half of \( 0 < t < 0.5 \) involves process “(re)loading”, while the second half of \( 0.5 < t < 1.0 \) involves process “unloading”, in terms of the shifted time \( t' \) of Eq. (7), with \( n_d = 1 \).

The slip/stick/open regions over an entire cycle are shown in Fig. 3, where \( x_{0} \) indicates the x-coordinate of a node of an undeformed initial state, and \( w \) is half of the block width, Fig. 1(a). The black contours in the figures represent \( v_{0}=0 \) and \( u_{0}=0 \). The left and right arrows in the figures indicate the backward and forward slips, Table 1. The gray contours in Fig. 3(a) represent \( |F_{i}/N_{i}| = \mu \), which corresponds to the critical condition between slip and stick status.

Fig. 3(a) shows that there is a contact region where \( |F_{i}/N_{i}| = \mu \) and there is no slip \( (v_{0}=0) \) during the unloading process \( (0.5 < t' < 1.0) \). Since a similar trend has already been reported by Ahn and Barber [26], this indicates that ABAQUS is capable of solving this type of problem. Note that the figure is a combined version of Fig. 9A and B of Ahn and Barber [26].

The figure also shows that there is spatial symmetry of the slip regions over the entire cycle for normal cyclic loading, but temporal repetition does not occur. A greater slip region is observed during reloading \( (0 < t' < 0.5) \). On the other hand, tangential cyclic loading results in biased slip velocity distributions over a cycle in the positive or negative \( x_{0}/w \) direction during the first or the second half, respectively, Fig. 3(b). They are symmetric with respect to \( x_{0}/w = 0 \), and occur with a time difference of \( 0.5 \text{ s} \), half of the period.

The frictional energy dissipation rates in Fig. 3(c) depict that it is the same for both \( Q_{0} < 0 \) and \( Q_{0} > 0 \) in tangential cyclic loading. However, it is not the same for \( P_{0} > P_{1} \) in normal cyclic loading, i.e. the reloading and unloading phases. There is higher frictional dissipation during reloading \( (0 < t' < 0.5) \) of normal cyclic loading than unloading \( (0.5 < t' < 1.0) \).

Fig. 3(a) and (b) shows that there are two different displacement modes during the cycle. For example, \( x_{0}/w = 0.4 \) in Fig. 3(b) exhibits slip motion in the positive and negative directions for \( 0.1 < t' < 0.25 \) and \( 0.6 < t' < 0.75 \), respectively. On the other hand, the node at \( x_{0}/w = 0.8 \) shows slip motion in the positive direction for \( 0.05 < t' < 0.25 \), then open status for \( t' > 0.45 \) until the end of the cycle. This implies that the node at

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**Table 1**

<table>
<thead>
<tr>
<th>State</th>
<th>Displacement or slip velocity</th>
<th>Reaction loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stick</td>
<td>( u_{0}=0, n_{i}=0 )</td>
<td>( N_{i}, &gt;0, -\mu N_{i} &lt; F_{i} &lt; \mu N_{i} )</td>
</tr>
<tr>
<td>Forward slip</td>
<td>( u_{0}=0, n_{i}=0 )</td>
<td>( N_{i}, &gt;0, F_{i} = -\mu N_{i} )</td>
</tr>
<tr>
<td>Backward slip</td>
<td>( u_{0}=0, n_{i}=0 )</td>
<td>( N_{i}, &gt;0, F_{i} = \mu N_{i} )</td>
</tr>
<tr>
<td>Open</td>
<td>( u_{0}=0 )</td>
<td>( N_{i}, F_{i} = 0 )</td>
</tr>
</tbody>
</table>

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**Fig. 3.** Slip/stick/open regions over a cycle for (a) \( P_{0}=500 \) and \( Q_{1}=0 \) MPa and (b) \( P_{0}=0 \) and \( Q_{1}=300 \) MPa. (c) Histories of frictional energy dissipation rates over a cycle for \( P_{0}=0 \) and \( Q_{1}=300 \), and \( P_{1}=900 \) and \( Q_{1}=0 \) MPa. (d) Frictional energy dissipation per cycle, for normal or tangential cyclic loading. \( P_{0}=1000 \) MPa unless otherwise specified.
and $f_{\text{obs}}$ observed in Fig. 4, i.e. there is a balance between displacement $x$ and counter displacement, for loading conditions shown in the figure, since the resultant load asymmetric distribution of the slip region is observed for all the phenomenon is observed for normal cyclic loading, Fig. 3(a).

The frictional energy dissipation per cycle for either normal or tangential cyclic loading is shown in Fig. 3(d) as a function of atime after a few cycles under the loading condition. A similar information in open status. Thus, the block reaches periodic steady state after a few cycles under the loading condition. A similar combination of frictional slip in contact status and elastic deformation in open status. Thus, the block reaches periodic steady state after a few cycles under the loading condition. A similar

4.2. Combined normal and tangential cyclic loading

In this section, normal and tangential loads are applied to the elastic block simultaneously with non-zero clamping load, i.e. $P_1 \cdot Q_1 \neq 0$ with $P_0 > 0$, Eq. (3). The angular speed $\omega$ is set as $2\pi$ rad/s, with the phase offset $\phi$ between 0 and $\pi$.

The slip/stick/open regions over a cycle, with $P_0 = 1000$ MPa and $\phi = 0$ are shown in Fig. 4 for select $P_1$ and $Q_1$. Severe asymmetric distribution of the slip region is observed for all the loading conditions shown in the figure, since the resultant load for $\phi = 0$ is a linear cyclic load which is inclined to the tangential direction. The figure shows that increasing $Q_1$ increases the slip region for the entire $x_0/w$ zone, while increasing $P_1$ increases the slip region only for positive $x_0/w$, but decreases the slip region for negative $x_0/w$. The steady state observation from Fig. 3 is also observed in Fig. 4, i.e. there is a balance between displacement and counter displacement, for $x_0/w = \text{const}$. In summary, when $\phi = 0$, frictional energy dissipation is not the same in the loading and unloading phases and shows spatially asymmetric distribution within a cycle.

The slip/stick/open regions over a cycle with $P_0 = 1000$ MPa and $\phi = \pi/2$ are shown in Fig. 5 for select $P_1$ and $Q_1$. $\phi = \pi/2$ obviously results in more evenly distributed slip regions than when $\phi = 0$, spatially and temporally. The figure also shows that increasing $P_1$ or $Q_1$ increases the slip region for the entire $x_0/w$ area. Another aspect for $\phi = 0$ is that the regions are affected in both time and spatial domains for different $P_1$ and $Q_1$. The previous observations on periodic steady state are also seen in Fig. 5 for $\phi = \pi/2$. In summary, $\phi = \pi/2$ also results in different frictional energy dissipation in the loading and unloading phases with spatially asymmetric distribution within a cycle.

The frictional energy dissipation rates for selected $\phi$ with $P_1 = 200, Q_1 = 275$ and $P_0 = 1000$ MPa are shown in Fig. 6(a). It is evident that there are two peaks over a cycle for all the $\phi$ in the figure. $\phi = 0$ is seen to result in large dissipation during the first half and then small dissipation during the second half, while $\phi = \pi$ results in the exact opposite. In addition, the peaks occur in the range $0.1 < t' < 0.15$ or $0.6 < t' < 0.65$. The frictional energy dissipation per cycle as a function of $\phi$ is shown in Fig. 6(b). To examine the contribution of each frictional energy dissipation, the time range is shifted by $-0.15$ s, i.e. “first half” and “second half” in the legend of Fig. 6(b) indicate the time ranges “A1” and “A2”, respectively in Fig. 6(a). Fig. 6(b) shows that maximum and minimum dissipation occur for $\phi = 0$ and $\pi/2$, respectively, while $\phi = 0$ and $\pi$ result in similar dissipation. The minimum and maximum are slightly off from $\phi = \pi/2$ and 0 by not more than $\pi/16$ rad. This is mainly due to the different frictional behaviors between the loading and unloading segments for normal cyclic loading, Fig. 3(c). The same frictional energy dissipation for the first and second half of a tangential cyclic loading cycle without normal cyclic loading ($P_1 = 0$), Fig. 3(c), does not appear in Fig. 6.
for combined tangential and normal cyclic loadings ($P_1 \neq 0$ and $Q_1 \neq 0$) for most of $\phi$.

Slip/stick/open regions over a cycle for $P_1 = Q_1 = 200$, $P_0 = 2000$ MPa and $\phi = \pi/2$ are shown in Fig. 7(a). A comparison of Fig. 7(a) with Fig. 5(c) for $P_1 = Q_1 = 100$ and $P_0 = 1000$ MPa shows that the area under the slip region and the time duration for the slip motions are the same for both $P_0 = 1000$ and 2000 MPa. Note that $P_1/P_0 = 0.1$ in both figures. The normalized tangential reactions $F_t/\mu P_0$ and slip velocities $\nu_s/\nu_s,\text{max}$ for both $P_0 = 1000$ and 2000 MPa are also compared in Fig. 7(b). The subscript "\( \pi,\text{max} \)" indicates that the slip velocities for both $P_0 = 1000$ and 2000 MPa are normalized by the maximum slip velocity for $P_0 = 1000$ MPa. Thus, they can be directly compared in the figure, where $F_t/\mu P_0$ graphs for $P_0 = 1000$ or 2000 MPa coincide with each other over the entire $x_0/w$ range. This means that $F_t$ increases by a factor of two as $P_0$ increases twice, from 1000 to 2000 MPa. The figure also shows that the slip velocity for $P_0 = 2000$ MPa is twice that for $P_0 = 1000$ MPa. Thus, the frictional energy dissipation per cycle will increase by a factor of $2^2 = 4$ as $P_0$ increases from 1000 to 2000 MPa for $P_1/P_0 = 0.1$. This implies that the frictional energy dissipation per cycle for different $P_0$ can be scaled for a given $P_1/P_0$ if a factor, namely $n_{P_0}$, is introduced as follows:

$$n_{P_0} = \left( \frac{P_0}{P_0,\text{ref}} \right)^2$$

where $P_{0,\text{ref}} = 1000$ MPa is a reference value of $P_0$. The frictional energy dissipation per cycle scaled with $n_{P_0}$ is shown as a function of normalized tangential cyclic loading $F_t/\mu P_0$ in Fig. 7(c) and (d). Frictional energy dissipation per cycle for the simple tangential cyclic loading in Fig. 3(d) is also scaled with $n_{P_0}$ and shown in the figure, denoted as "$P_1/P_0 = 0$". The figures show that $n_{P_0}$ scales the frictional energy dissipation per cycle with respect to the
Fig. 7. (a) Slip/stick/open regions over a cycle with $P_1 = Q_1 = 200$, $P_0 = 2000$ MPa and $\phi = \pi/2$. The thin contours are for $P_1 = Q_1 = 100$ and $P_0 = 1000$ MPa. (b) Normalized tangential reactions and slip velocities at $t' = 0.25$ s. Scaled frictional energy dissipation per cycle as a function of normalized $Q_1$ for (c) $\phi = 0$, and (d) $\phi = \pi/2$. $P_0$ in GPa for figures (c) and (d).

Fig. 8. (a) Slip/stick/open regions over a cycle with $P_1 = Q_1 = 200$ MPa for $h = 2.5$ mm. The thin contours are for $h = 10$ mm. (b) Frictional energy dissipation per cycle as a function of $h$ for $P_1 = 300$ MPa, with dotted curves representing $W_{\mu}/h^{1.54}$. For the figures, $\phi = \pi/2$ and $P_0 = 1000$ MPa.

normalized $Q_1$ for the same $P_1/P_0$ onto one curve. However, it should be remembered that $W_{\mu}/P_0$ is not dimensionless. The figures also show that the normalized $Q_1$ has more significant impact on frictional energy dissipation per cycle than the normalized $P_1$ when $P_1/P_0$ is relatively low, and vise versa for relatively high $P_1/P_0$. In addition, the figures show, together with Fig. 6(b), that frictional energy dissipation is higher for $\phi = 0$ than $\phi = \pi/2$ for given normalized $P_1$ and $Q_1$, for all the normalized $Q_1$ within the range shown in the figures.

4.3. Studies on block height and angular speed

In this section, the effect of the block height $h$, angular speed $\omega$, and friction coefficient $\mu$ on frictional energy dissipation is investigated for combined normal and tangential cyclic loadings with $P_0 = 1000$ MPa and the phase offset $\phi = 0$ or $\pi/2$.

To examine the block height, the element size for the mesh shown in Fig. 1(b) is kept constant at $\Delta x = \Delta y = 0.125$ mm. The slip/stick/open regions over a cycle for $h = 2.5$ mm is shown in Fig. 8(a). Fig. 5(b) for $h = 10$ is re-plotted in the figure as thin gray contours for comparison. The figures show that lower $h$ results in a significantly smaller slip region with more open region at the outer portion, while there is no significant change for the slip region in the time domain, $t'$. This implies a reduction in frictional energy dissipation for lower $h$ at a given loading condition. The trend can be seen from Fig. 8(b). The figure shows that frictional energy dissipation per cycle has an approximate relation with the block height of $W_{\mu} \propto h^{1.54}$ from curve fits for relatively high $h$, i.e. $h \geq 5$, where the relation is plotted as dotted curves for different $Q_1$.
Next, Eq. (3) is modified to consider different angular speeds between normal and tangential cyclic loadings as follows:

\[ P(t) = P_0 + P_1 \cdot \sin(\omega_P t - \phi) \]  
\[ Q(t) = Q_1 \cdot \sin(\omega_Q t) \]  

where \( \omega_P \) and \( \omega_Q \) are the angular speeds for normal and tangential cyclic loading, respectively. In the following analysis, \( \omega_P = 4\pi/3 \) and \( 4\pi \) are selected, with \( \omega_Q = 2\pi \) rad/s. The time step \( \Delta t \) is selected as the smaller of \( 0.01 \cdot (2\pi/\omega_P) \) and \( 0.01 \cdot (2\pi/\omega_Q) \) s. Frictional energy dissipation rates for different \( \omega_P \) are shown in Fig. 9(a). Eq. (10) with \( \omega_Q = 4\pi/3 \) rad/s, \( P_1 = Q_1 = 200 \) and \( P_0 = 1000 \) MPa is also plotted in the figure. Frictional energy dissipation rate in the figure shows that \( \omega_P = 4\pi \) results in temporally repetitive frictional energy dissipation within one second, the period of \( Q(t) \). This trend is similar to simple tangential cyclic loading, Fig. 3(c). On the other hand, \( \omega_P = 4\pi/3 \) results in temporally repetitive frictional energy dissipation within 3 s. It can be observed for \( \omega_P = 4\pi \) in Fig. 9(a) that there are two positive and one negative peaks of \( Q(t) \) during the first cycle of \( P(t) \), while there are two negative and one positive peaks of \( Q(t) \) during the second cycle of \( P(t) \). For \( \omega_P = 4\pi \), we note from Eq. (10) that \( P(t) \) completes the unloading and loading processes while \( Q(t) \) undergoes half of the cycle. This can be visualized by the displacement loci of the top center node \((x=0, y=h) \) in Fig. 1(a) of the block for \( \omega_P = 2\pi \) and \( 4\pi \), Fig. 9(b). The arrows in the figure represent the direction of motion of the node. It should be recalled that simple normal cyclic loading produces different frictional energy dissipation during unloading and reloading processes within a cycle, Fig. 3(c). Thus, the temporally repetitive frictional energy dissipation rate in Fig. 9(a) is due to the fact that the normal cyclic unloading and reloading processes are evenly distributed over positive and negative loading of \( Q(t) \).

![Fig. 9](image-url)

**Fig. 9.** (a) Loading condition (top) and frictional energy dissipation rates for selected \( \omega_P \) (bottom). (b) Displacement loci at the top center node over a cycle for selected \( \omega_P \) and \( \phi \). Slip/stick/open regions over a cycle for (c) \( \omega_P = 4\pi/3 \) and (d) \( \omega_P = 4\pi \) rad/s. For the figures, \( \phi = 0, \omega_Q = 2\pi \) rad/s, \( P_1 = Q_1 = 200 \), and \( P_0 = 1000 \) MPa, unless otherwise specified.

4.4. Combined loads for ultrasonic welding

A typical ultrasonic welding system is illustrated in Fig. 10(a). The process involves a stationary clamping load and a tangential cyclic load, i.e. \( P_1 = 0, P_0 > 0 \) and \( Q_1 \neq 0 \) for Eq. (3). It is known that ultrasonic welding quality is closely related to frictional heat, and the heat is related to several parameters including static clamping load, \( P_0 \), and tangential cyclic loading, \( Q_1 \) [9,10]. Thus, it can be assumed to produce a good weld joint when gross slip occurs between the workpieces to generate a significant amount of frictional shear work at the interface. And, the loading condition for ultrasonic welding is assumed to exceed the static limit of Eq. (4) for good quality welds, i.e. there should be time \( t \) when the relation \( Q(t) > \mu \cdot P(t) \) is satisfied. This means that the cases examined in the preceding sections correspond to low-quality welding conditions. The results thus provide information that will enable low-quality welding conditions to be avoided during ultrasonic welding.
A loading case with \( P_1 = 0, P_2 > 0 \) and \( Q_1 = Q_n \pm q_t \) of Eq. (3) is considered for further discussion, and is shown as case “a” in Fig. 10(b). \( Q_n \) (circular mark) and \( q_t \) (horizontal error bar) are the nominal value and uncertainty of tangential cyclic loading, respectively, and they are positive with \( Q_n > q_t \). The curve for \( \phi = \pi/2 \) or line for \( \phi = 0 \) in the figure that distinguishes local slip from gross slip is from Fig. 2(b). Case “a” in the figure can be written as
\[
Q_n > \mu \cdot P_0
\]
\[
Q_n - q_t < \mu \cdot P_0
\]
Eq. (11b), i.e. by reducing the right hand side of Eq. (11b), i.e. by reducing the clamping load \( P_0 \) or the friction coefficient \( \mu \). However, care should be taken in reducing \( P_0 \) since it may affect frictional energy dissipation and frictional shear work done at the interface. \( \mu \) may be reduced by improving the workpiece surface finish, which will increase manufacturing cost. The case may also be improved by increasing the left hand side of Eq. (11b), i.e. by reducing \( q_t \) or increasing \( Q_n \). However, both options will involve significant design changes for the tangential cyclic loading device.

A combined normal and tangential cyclic loading improves the situation, case “c” in the figure. This would involve some design modifications for clamping. As case “b” of the figure shows, gross slip can be obtained for \( \phi = 0 \) with a lower \( Q_n \) than for the default loading, case “a”. It should be recalled that combined loading results in asymmetric slip regions, Figs. 4 and 5. Thus, a workpiece may drift away from the original starting point as combined cyclic loading of Eq. (3) is applied due to the asymmetric slip/stick behavior, rather than the piece oscillating about the point. The drift implies that the workpieces may be joined, or assembled, with offset from the desired dimensions. However, as Fig. 9(d) shows, such asymmetric slip regions do not occur when normal and tangential cyclic loading is applied at different frequencies, thereby reducing the possibility of drifting.

5. Conclusions

Contact between an elastic block and a frictional rigid surface under normal and/or tangential cyclic loading, combined with a constant normal loading was studied using ABAQUS, with a quasistatic assumption.

First, the effect of separately applying normal and tangential cyclic load with a mean normal load was investigated. The results indicate that normal loading has greater impact on the frictional energy dissipation per cycle than tangential loading. The results also show that normal cyclic loading results in a spatially symmetric slip/stick distribution throughout a cycle. On the other hand, unloading and reloading segments of normal cyclic loading show different slip/stick distributions. Tangential cyclic loading results in spatially alternating slip/stick distribution over a cycle, with the alternating slip regions being symmetric to each other and occurring with a time difference corresponding to half of the period.

The results indicate that combined cyclic loading results in asymmetric slip/stick distribution over a cycle regardless of the phase offset, if the loads are applied at the same frequency. A normal load scaling factor for frictional energy dissipation was developed from the results, and indicates that frictional energy dissipation will increase in proportion to the square of the increment of mean normal loading for given normalized normal and tangential cyclic loadings.

Symmetric slip/stick distribution over a cycle when the system is subjected to a combined cyclic loading can be obtained if the loads have different frequencies, and if the frequencies are selected such that both normal reloading and unloading segments occur during each positive or negative tangential cyclic loading. A thinner block results in less frictional energy dissipation per cycle for a given loading condition.

The quasistatic analysis provides information that can serve as a guide for selecting desirable ultrasonic welding conditions. The studied parameters include average normal loading, and nominal value and the uncertainty of tangential loading. Application of a combined normal and tangential cyclic loading is also examined, and the results indicate that a combined cyclic loading can be useful in ensuring good quality welds.

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References
