Technical Note

Effect of the volume of a functionally graded material layer on frictionally excited thermoelastic instability

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Abstract

A finite element model is used to identify the effect of the volume of a functionally graded material (FGM) on thermoelastic instability (TEI). An optimal FGM volume that exhibits the highest critical speed was found to exist. Beyond the optimal FGM volume, the critical speed is much lower than that of a homogeneous steel layer. For all FGM volumes, the performance against TEI is dominant for the same nonhomogeneous parameter, which determines the compositional shape of the material property grading in the FGM. In addition, the thermal conductivity of the frictional material and the modulus of elasticity of the FGM were found to have the most significant impact on an increase in the critical speed.

1. Introduction

Any device that is subjected to frictional heating between sliding interfaces when the relative sliding speed is critically high usually generates hot spots during the process of sliding. This frictional instability, known as thermoelastic instability (TEI), is associated with several technological problems, such as the "hot judder" vibration issue in automotive brake disks [1,2].

Experimental evidence of the hot spotting phenomenon was first reported by Parker and Marshall [3] in tread-braked railway wheels, and an explanation of the phenomenon in terms of TEI was first advanced by Barber [4]. Theoretical investigations of TEI were pioneered by Burton [5], who introduced the concept of a critical sliding speed for instability. Lee and Barber [6] applied Burton's method to the problem of a layer sliding between two stationary half planes. Such an approach was regarded as an idealization of brake disk sliding between two frictional pads. Early attempts at numerical simulations of TEI in a disk brake were conducted by Kennedy and Ling [7]. Zagrodzki et al. [8] described a two-dimensional direct numerical simulation method for a three-layer system that was somewhat similar to Lee and Barber's layer model. In a different approach, Yi et al. [9] attempted a finite element implementation of Burton's perturbation technique and obtained a generalized linear eigenvalue problem for the growth rate. As an alternative to the use of conventional ferrous materials for enhancing the performance of a frictional system, Jang and Ahn [10] employed functionally graded materials (FGMs) in a frictional sliding system and demonstrated their feasibility to produce better performance against TEI [11−13]. FGMs, defined as a structure where a smooth gradation exists between two or more nonhomogeneous materials, have the heat and corrosion resistance of a ceramic and the mechanical strength of metal. Consequently, FGMs allow for a higher operating temperature to be reached. This in turn increases both the thermal efficiency of a system and the bonding strength along the coating substrate interfaces [14,15]. The nonhomogeneous properties of an FGM can be further tailored so as to reduce the thermal and residual stresses in an elevated temperature environment [16,17].

Recent investigations of TEI in FGMs [3] have demonstrated that better performance can be obtained when the nonhomogeneous parameter, which determines the compositional shape of the material property grading in FGMs, reaches a certain value. At such a value, a limited volume of the coating layer of the FGM on the core steel material slides against a conventional frictional material. Further investigations of the model have shown some unexpected sensitivity to the exact distribution of the FGM composition. Thus, additional research is needed to determine whether the performance against TEI will be better if the volume of the FGM coating increases. An increase in the volume of the FGM coating layer means that a portion of the ceramic material will increase, leading to an enhancement in its role as a thermal barrier. On the other hand, it is possible that specific mechanical properties, such as the modulus of elasticity, will degrade. Furthermore, the behavior of FGMs is quite complex because of their graded properties.

In this paper, the effect of the volume of the FGM coating layer on TEI is investigated. Performance estimators of TEI, such as the
critical speed of the system, are evaluated and the optimal volume of the FGM coating layer is determined.

2. Description of the numerical model

Since the current analysis is an extension of previous research, most of the model described here is derived from the work of Jang and Ahn [10]. The numerical model representing the sliding contact system is shown in Fig. 1. Two layers Ω₁ and Ω₃ of the friction material move in the plane with a relative sliding speed, \( V \), relative to \( Ω₂ \), which composed of steel (\( Ω_a \)) and an FGM coating layer (\( Ω_l \)). The thicknesses of \( Ω₁ \) and \( Ω₂ \) are both \( h \). The total thickness of layer \( Ω₂ \) is \( 2a+2l \), where \( 2a \) and \( 2l \) are the thickness of the core material and the thickness of the FGM coating layer, respectively. The model is geometrically symmetric with respect to the axis, \( y = h + a + l \).

The thermoelastic problem is assumed to be cyclically symmetric in the sliding direction \( x \) with wavelength \( L = C/m \), where \( C \) is the perimeter of the disk, and \( m \) is the wavenumber. Each layer of the system is elastic and thermally conductive. Although the proposed model has a simple layered structure, it may address important technical issues related to the practical applications of brakes or clutch disks. For example, Lee and Barber [11] successfully determined the characteristics of TEI in a homogeneous layer structure. Such findings are frequently applied to the design of brakes and clutch systems.

3. Material properties in functionally graded material

The material properties of the model are mainly determined by the homogeneous frictional material, the steel in the core, and the FGM in the coating layer. The properties vary along the height dimension in the FGM domain. The FGM is designed to have a smooth property grading and thus, the variation in the thickness of the volume fractions of the two materials is continuous. In this analysis, we combined two materials, a steel and a ceramic, to form the FGM. The properties of the steel, the ceramic material, and the frictional material are shown in Table 1. Note that the material properties in the current model are almost the same as those in a previous model [10], except that the frictional material had a low thermal conductivity.

In the FGM domain, the material properties are given by [3]

\[
P = \frac{P_c + (P_r - P_c) \left( \frac{h + 2a + 2l - y}{l} \right)}{2}, \quad h + 2a + l < y < h + 2a + 2l
\]

where \( P, P_c, \) and \( P_r \) are the material properties which vary with the height, the properties of the steel, and the properties of the ceramic, respectively. The parameter \( P \) can represent the elastic modulus, the thermal expansion coefficient, the thermal conductivity, and the thermal diffusivity, while \( \alpha \) denotes the nonhomogeneity parameter of the FGM. According to the range of \( \alpha \), the composition of the FGM can vary. We choose \( \alpha \geq 1 \) for ceramic-rich FGM, which exhibits better performance with respect to TEI [10] when compared to a metal-rich FGM.

4. The finite element model

A transient coupled thermo-mechanical analysis is used for the frictionally excited thermoelastic instability problem. In the thermal analysis, a special treatment for the convective term is

### Table 1

<table>
<thead>
<tr>
<th>Properties</th>
<th>Frictional material</th>
<th>Steel</th>
<th>Ceramic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus (GN/m²)</td>
<td>0.3</td>
<td>200</td>
<td>151</td>
</tr>
<tr>
<td>Thermal expansion (µC⁻¹)</td>
<td>14</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Thermal conductivity (W/m°C)</td>
<td>0.241</td>
<td>42</td>
<td>2.2</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.12</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Thermal diffusivity (µm/s²)</td>
<td>0.474</td>
<td>11.91</td>
<td>0.623</td>
</tr>
</tbody>
</table>

Fig. 1. Model with a frictional material and an FGM layer.
required due to the relative sliding speed between the layers. Furthermore, the nonhomogeneity of the FGM should be considered in the detailed material modeling and the corresponding numerical algorithm. Such issues were previously considered on a theoretical and numerical basis. Thus, in this analysis, we simply highlight the differences between the previous analysis and the present one.

In a previous analysis [3], the FGM layer was discretized into multiple layers with constant properties. The current FGM layer is modeled with a continuously varying composition and properties that vary in the thickness direction. To implement the model, implicit user-defined material subroutines in ABAQUS [18], such as UMAT, UMATHT, and UEXPAN, are developed and used in the FE calculations.

The interaction between the frictional sliding surfaces of the frictional material and the FGM layer generates strong temperature gradients in the skin layer of the frictional material. Thus, when compared to the element size in the FGM layer, a much smaller element size is employed in the skin layer of the frictional material.

In the simulation procedure, the thermal and elastic contact problems are solved sequentially with respect to time. In the thermal analysis, the heat flux at time \( t + n \Delta t \), where \( t \) is the time at which the solution is known, and \( n = 1, \ldots, N \) are approximated by \( q_i^{n+M} = fVp_\tau \), where \( p_\tau \) is the contact pressure at time \( t \). After calculating \( T^{n+NM} \), the elastic contact problem at the time \( t + N \Delta t \) is solved.

5. Results

The performance on TEI is characterized by estimating the critical speed [5]. It is generally accepted that the instability is determined by a perturbation of the contact pressure. Such a perturbation can be represented by an exponential in the form of \( e^{-bt} \) where \( b \) is the growth rate. When the growth rate is negative or positive, the system is unstable or stable, respectively. The speed at which the growth rate is zero (neutral stable condition) is then defined as the critical speed, \( V_{cr} \). To find the critical speed numerically, we calculate the growth rates at several speeds and find the speed at which the growth rate is zero. In these calculations, the dimensionless critical speed \( V_{cr}^* \) is defined as

\[
V_{cr}^* = \frac{V_{cr}(a+l)}{k_\Omega} \quad (2)
\]

where \( k_\Omega \) is the thermal diffusivity of \( \Omega_s \) and \( a+l \) is the half-thickness of the central layer \( \Omega_c \).

5.1. Effect of the FGM volume

To investigate the effect of the volume of the FGM on TEI, the volume of the FGM coating layer \( \Omega_c \) was varied from 0% to 100% of the total thickness of layer \( \Omega_s \). The volume percent of FGM layer, \( w_m \), is defined as

\[
w_m = \frac{L}{L + a} \times 100(\%)
\]

When \( w_m \) is 100%, the symmetrical half layer in \( \Omega_s \) is composed of the FGM. When \( w_m \) is zero, the layer is composed of steel.

Shown in Fig. 2 is the dimensionless critical speed with respect to the wavenumber for different values of the FGM volume when the nonhomogeneous parameter is 7. It is found that the critical speed is highest when the volume percent is 20%. If the volume of FGM is increased further, the critical speed decreases. When the volume percent is greater than 40%, the critical speed is less than that obtained at 0% for a homogeneous steel layer. The critical speed is determined from information on the growth rate. The growth rate is almost linear over a small sliding speed range, but behaves nonlinearly at a high sliding speed.

To confirm the fact that the critical speed is highest at an FGM volume of 20%, we extend the range of the FGM volume and obtain the minimum critical speed among all critical speeds at a given FGM volume. The minimum critical speed with respect to the volume of FGM when the nonhomogeneous parameter is fixed at 7 is shown in Fig. 3. It is evident that the critical speed is highest at an FGM layer volume of 20%.

It is interesting that an increase in the volume of FGM causes degradation in the performance against TEI. To validate these
findings, the deformation of layers with different FGM volume percentages at a specific time after engagement was examined; the results are shown in Fig. 4. Each dataset was obtained at a fixed time. The antisymmetric deformation is more severe as the volume of FGM increases.

5.2. Effect of the nonhomogeneous parameter

The behavior of the critical speed with respect to the nonhomogeneous parameter for different FGM volume percentages is plotted in Fig. 5. At each FGM volume percent, the lowest critical speed is determined when the nondimensional parameter \( m(a + t) \) is fixed to a specific value. As the nonhomogeneous parameter increases from \( a = 1 \), the critical speed increases, approaches a maximum value at \( a = 7 \), and then decreases toward a finite limit. What is more interesting is that the critical speed is always maximum when the nonhomogeneous parameter \( a \) is 7, regardless of the variation of the FGM volume.

In a previous analysis [3], the maximum critical speed was found to occur at \( a = 6 \), which is slightly different from the current result. Such a difference may come from the fact that, in this work, the thermal conductivity of the frictional material is lower than that used in the previous research [3].

5.3. Effect of material properties

The above results were obtained for a particular set of material properties. Thus, the next question is whether or not relatively small changes in the material properties can have a significant effect on the optimality of the critical speed. To answer this question properly, we must first identify the realistic range of material properties for actual applications. We focus on the material properties in the frictional material and the properties of the ceramic in the FGM. Specifically, the properties under consideration are the modulus of elasticity, the thermal conductivity, and the thermal expansion coefficient, since these properties have the greatest impact on the critical speed.

The material properties of the frictional material were taken from Refs. [19,20], while the properties of the ceramic in the FGM were derived from Refs. [14,21]. Practical material property ranges for the frictional material include 0.3–0.5 GN/m² for the modulus of elasticity, 0.24–1 W/m°C for the thermal conductivity and 5–14 μ°C for the coefficient of thermal expansion. For the ceramic material, the modulus of elasticity ranges from 50 to
151 GN/m², the thermal conductivity ranges from 0.7 to 2.2 W/m°C, and the coefficient of thermal expansion ranges from 8 to 10 μC⁻¹. To estimate the effect of the material properties on TEI, a material property is selected and a value within the appropriate range is selected while the other properties are fixed as the current set of material properties. For example, when the thermal conductivity in the frictional material is selected as 0.5 W/m°C, the other material properties in the frictional material and the material properties of the FGM are fixed to the current set of properties. We then calculate the minimum critical speed.

A series of results for the minimum dimensional critical speed $V_{n}^{*}$ with respect to the volume percent of FGM ($w_{m}$) for different modulus of elasticity values, thermal conductivities, and thermal expansion coefficients of the frictional material is shown in Figs. 6, 7, and 8, respectively. The critical speed is highest at an FGM volume percent of 20% for all sets of material properties. If the modulus of elasticity, the thermal conductivity, and the thermal expansion coefficient of the frictional material are increased, the critical speed increases. When the thermal conductivity of the frictional material is increased, the critical speed at an FGM volume percent of 40% exhibits a slight increase.

A series of results for the minimum dimensional critical speed $V_{n}^{*}$ with respect to the volume percent of FGM ($w_{m}$) for different modulus of elasticity values of the ceramic material in the FGM. Circles, squares, and triangles represent cases where the elastic modulus is 50, 100, 151 GN/m², respectively.

A series of results for the minimum dimensional critical speed $V_{n}^{*}$ with respect to the volume percent of FGM ($w_{m}$) for different thermal expansion coefficients of the frictional material. Squares, circles, and triangles represent cases where the thermal expansion coefficient is 5, 8, and 14 μC⁻¹, respectively.

FGM volume percent of 20% for all sets of material properties. If the modulus of elasticity, the thermal conductivity, and the thermal expansion coefficient of the frictional material are increased, the critical speed increases. When the thermal conductivity of the frictional material is increased, the critical speed at an FGM volume percent of 40% exhibits a slight increase.

A series of results for the minimum dimensional critical speed $V_{n}^{*}$ with respect to the volume percent of FGM ($w_{m}$) for different modulus of elasticity values, thermal conductivities, and thermal expansion coefficients of the frictional material is shown in Figs. 9, 10, and 11, respectively. For all sets of material properties, the critical speed is highest at an FGM layer volume of 20%. If the modulus of elasticity, thermal conductivity, and thermal expansion coefficient of the FGM decreases, the critical speed increases.
such difficulties will now be explained. While some simulation results have been obtained, it is difficult to observe when compared to the case of a homogeneous steel layer. For a larger FGM volume, an adverse effect could be determined by a thermal or contact pressure disturbance that thermal expansion coefficient is 5, 8, and 14 $\mu \text{C}^{-1}$, respectively. Circles, squares, and triangles represent cases where the thermal expansion coefficient is 5, 8, and 14 $\mu \text{C}^{-1}$, respectively.

6. Discussion

An increase in the FGM volume induces a significant improvement in the frictional system with regard to TEI, particularly at small FGM volume percentages, i.e., a thin coating layer. Such a result is consistent with the findings of the previous research [10]. However, for a larger increase in the FGM volume, an adverse effect could be observed when compared to the case of a homogeneous steel layer. While some simulation results have been obtained, it is difficult to rigorously analyze TEI in the FGM layer structure. The reason for such difficulties will now be explained.

For the frictionally excited TEI problem, the stability is generally determined by a thermal or contact pressure disturbance that migrates with respect to the two contacting interfaces [5]. Specifically, the migration speed of a poor thermal conductor (large FGM volume) is larger than that of a good thermal conductor (small FGM volume). As a result, the dominant perturbation in the good thermal conductor migrates fast, which increases the thermal expansion at a given perturbation of the heat input and decreases the critical speed. While the disturbance at the interface is affected by the thermal properties, thermoelastic deformation of the finite layer is mainly dependent on the mechanical properties. For a larger FGM volume, the modulus of elasticity is smaller than that for a smaller FGM volume. Thus, thermal deformation becomes larger, as shown in Fig. 4. When the inclusion of an FGM is considered, the grading of the material properties in the FGM makes it even more difficult to explain the behavior of TEI.

We found that the maximum critical speed always occurs at the same nonhomogeneous parameter, regardless of the FGM volume. It is debatable whether the current FGM composition is responsible for such a finding, or whether other FGM compositions with a non-power law relationship may behave differently. This problem may require another comprehensive study to determine the optimal FGM composition with various material properties and grading shapes.

The thermal conductivity of the frictional material and the modulus of elasticity of the FGM were found to have the most profound effect on an increase in the critical speed. This could be a possible material design guideline for enhancing the performance against TEI.

7. Conclusion

This investigation shows that a small FGM coating layer volume enhances the performance against TEI, while a larger FGM coating layer volume may degrade the frictional system by inducing severe thermoelastic instability. Specifically, the critical speed reaches a maximum value when the FGM volume is 20% of the half-thickness of the layer. The critical speed decreases below that of homogeneous steel when the FGM volume is greater than 40%. For all FGM volume percentages, the highest critical speeds are obtained at the same nonhomogeneous parameter. In addition, the thermal conductivity of the frictional material and the modulus of elasticity of the FGM have the most profound effect on an increase in the critical speed.

References


