ERRATUM TO “ON THE HÖLDER CONTINUITY OF SOLUTIONS OF A CERTAIN SYSTEM RELATED TO MAXWELL’S EQUATIONS”

KYUNGKEUN KANG* AND SEICK KIM†

The main purpose of this erratum is to correct Lemmas 2.4 and 2.5 in [2] and present their proofs. We also take this opportunity to rectify some flaws caused by those incorrectly stated lemmas.

First we make a correction to the definition of \(D(\Omega)\) (page 88, line 25) as follows:

\[ D(\Omega) = D(\Omega; \mathbb{R}^3) = \{ f \in C^\infty(\Omega; \mathbb{R}^3) : f \text{ is compactly supported}, \nabla \cdot f = 0 \}. \]

In Theorem 2.1, Theorem 2.2 and the other related part of article, \(f \in H^q_\text{loc}(\Omega)\) should read \(f \in \mathcal{H}^q(\Omega)\) and \(\|f\|_{L^s(B)}\) should read \(\|f\|_{L^s(\Omega)}\).

Then, Lemma 2.4 and Lemma 2.5 should be corrected as follows:

**Lemma 2.4.** Let \(\Omega \subset \mathbb{R}^3\) be an open set and assume \(f \in D(\Omega)\). Then there exists \(g \in C^\infty(\Omega)\) such that \(\nabla \cdot g = f\) and \(\nabla \cdot g = 0\) in \(\Omega\). Moreover, we have

\[ \|\nabla g\|_{L^p(\Omega)} \leq C(p) \|f\|_{L^p(\Omega)} \text{ for } 1 < p < \infty. \]

**Proof.** We define \(g := -\nabla \times N(f)\), where \(N(f)\) is the Newtonian potential of \(f\) over \(\Omega\) (see e.g. [1, pp. 51] for definition). Then from the following vector identity,

\[ \nabla \times (\nabla \times F) = \nabla (\nabla \cdot F) - \Delta F, \quad (2.4) \]

we find

\[ \nabla \times g = -\nabla \times (\nabla \times N(f)) = \Delta N(f) - \nabla (\nabla \cdot N(f)) = f - \nabla (\nabla \cdot f) = f. \]

Also, by Calderón-Zygmund theory (see e.g. [1, Theorem 9.9]), we find \(\|\nabla g\|_{L^p(\Omega)} \leq C \|f\|_{L^p(\Omega)}\). Clearly, we have \(\nabla \cdot g = 0\).

**Lemma 2.5.** Suppose \(F \in C^\infty(\overline{B}; \mathbb{R}^3)\) satisfies \(\nabla \times F = 0\) in \(B\). Then there exists \(\varphi \in C^\infty(B)\) such that \(\nabla \varphi = F\) and \(\int_B \varphi = 0\). Moreover, we have \(\|\varphi\|_{L^3(B)} \leq C \|F\|_{L^3(B)}\).

**Proof.** Let \(\varphi\) be a solution to

\[ \begin{cases} \Delta \varphi = \nabla \cdot F & \text{in } B, \\ \frac{\partial \varphi}{\partial n} = F \cdot n & \text{on } \partial B. \end{cases} \]

By subtracting a constant, we may assume \(\int_B \varphi = 0\).

Denote \(\omega := \nabla \varphi - F\). We have \(\nabla \cdot \omega = 0\), \(\nabla \times \omega = 0\) in \(B\) and \(\omega \cdot n = 0\) on \(\partial B\). Therefore \(\omega \equiv 0\) in \(B\). We have thus shown that \(\nabla \varphi = F\) in \(B\). Since \(\int_B \varphi = 0\), Poincaré inequality implies \(\|\varphi\|_{L^3(B)} \leq C \|F\|_{L^3(B)}\).

Finally, we should make a slight change in the proof of Theorem 2.1.

On page 91, line 2, \(f \in H^q_\text{loc}(\Omega) \cap C^\infty(\Omega)\) should read \(f \in D(\Omega)\).

On page 91, line 7–10 should be replaced as follows:

Since \(f \in D(\Omega)\), we conclude from Lemma 2.4 that there exists a smooth vector \(g\) such that \(f = \nabla \times g\) in \(\Omega\). By subtracting a constant vector, we may assume that \(\int_B g = 0\). Then Sobolev-Poincaré inequality implies

\[ \|g\|_{L^{q^*}(B)} \leq C \|\nabla g\|_{L^3(B)} \leq C \|f\|_{L^s(\Omega)}, \quad q^* = nq/(n-q) > n. \quad (2.5) \]

*Department of Mathematics, University of British Columbia, 121-1984 Mathematics Road, Vancouver, B.C., Canada V6T 1Z2 (kkang@math.ubc.ca)
†Mathematics Department, University of Missouri, Columbia, MO 65211 USA (seick@math.missouri.edu)
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REFERENCES
