# The Value of an Informed Bidder in Common Value Auctions*

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**Abstract**

I compare two information structures in a common value first-price auction with two bidders: In one, each of the two bidders knows only his own signal about the value of the object, and in the other, one of the bidders learns his opponent's signal as well. Gaining the additional information in the second information structure makes the informed bidder worse off if the value is submodular in the bidders' signals and better off if it is supermodular. If the value is supermodular, then the seller's revenue tends to be lower with the informed bidder than without.

*JEL classification:* C72; D44; D86
*Keywords:* Common value; First-price auction; Informed bidder

## 1 Introduction

Suppose two bidders compete in a first-price auction for the right to operate in an oil tract, which consists of two areas that may contain oil. Each bidder is allowed to survey either area before bidding and draw signals about the size of the oil deposit. Bidder 2 has an efficient survey technology so he can survey both areas without incurring much cost. Bidder 1, a less efficient surveyor, can only survey one area. Assuming that a survey on a given area reveals same information, bidder 2 can survey either both areas

*I thank Yeon-Koo Che for discussion and comments on this paper, Hana Choi for proof-reading, and an associated editor and two referees for very thoughtful comments. This research has been supported by the "Fund for Supporting Basic Science Research" of the College of Business and Economics at Yonsei University.

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or just one but different from the one surveyed by bidder 1.[1] The former option will give bidder 2 an informational advantage over bidder 1 since he will then learn the value of the auctioned object (the oil deposit under both areas) as well as his rival's signal. Will such informational advantage always translate into a higher payoff for bidder 2? How will the seller's revenue be affected by the informational advantage? The current paper studies these questions.

Information plays an important role in many auction environments, and the attempt to understand it is as old as the seminal and classic contribution by Milgrom and Weber (1982a). They study a general symmetric (more precisely, exchangeable) informational environment including common values, and study the effect of revealing additional information on revenue and bidder payoffs. Their linkage principle states that publicly revealing additional information that affects all bidders symmetrically can only raise the revenue of the seller. That the information affects all bidders symmetrically limits the application of the result; for instance, in the situation described above, the learning of information is asymmetric between the two bidders. Extension of the analysis to asymmetrically impacting information can be seen in Milgrom and Weber (1982b) who study the effect of extra information on an informed bidder's payoff when the other bidder only has public information. In contrast, Campbell and Levin (2000) consider the bidders who are initially *asymmetrically informed*, and investigate how one bidder learning the other's information affects the seller's revenue, finding out that it increases unambiguously. They interpret this finding as an extension of the linkage principle to the case where the additional information affects bidders asymmetrically. Unlike these works, Kim and Che (2004) and Fang and Morris (2006) consider a private value setup and find out that some group(s) of bidders learning *each other*'s information can reduce the seller's revenue.

In this paper, I investigate the value of asymmetrically impacting information following the works of Campbell and Levin (2000) (henceforth, CL), but with more general assumptions on signal distribution. To this end, I consider two information structures in a first-price auction with common values: (1) the standard information structure (Milgrom and Weber, 1982a) where each of the two bidders knows only his own signal about the value of the object, and (2) the alternative information structure in which one of the bidders learns his rival's signal as well. The two information structures are then

[1] Clearly, it is suboptimal for bidder 2 to survey only one area, which is the same that is being surveyed by bidder 1.

compared in terms of the seller's revenue and the informed bidder's payoff. The learning of information by one bidder has an unambiguous effect on the other bidder, for the latter's payoff declines to zero. The effect on the former bidder is nontrivial, however. The key is how the bidders' common assessment of the object's value depends on their signals, in particular whether the common value is supermodular or submodular in the signals. The results are most clear-cut when signals are identically and independently distributed (IID). In this case, a bidder's payoff increases (resp. decreases) with his learning of the rival's signal if the value of the auctioned object is supermodular (resp. submodular) in the signals. Combining the observations, if the value is submodular, both bidders' payoffs fall and the seller's revenue rises as the bidder becomes more informed. By contrast, if the value is supermodular, two bidders' payoffs change in the opposite direction, which makes it unclear what information structure yields higher revenue for the seller. However, if the value is sufficiently supermodular, then the informed bidder's payoff becomes so large that the seller's revenue falls with the additional information.

This contrasts with the CL's finding that with affiliated and binary signals, as one bidder learns his rival's signal, both his payoff and seller's revenue increase. It turns out that the CL's setup falls into a special case in which the the common value is modular (i.e., the cross partial derivative of the value with respect to the signals is zero). Generalizing the CL's setup to allow for the sub/supermodular values, I obtain a result similar to that in the IID case: As the degree of supermodularity increases, one bidder's payoff is more likely to rise by learning his rival's signal while it can fall when the value is highly submodular. Also, if the value is highly supermodular, then the seller's revenue falls as one bidder learns the other's signal. To relate and contrast this result to the linkage principle of Milgrom and Weber (1982a), consider a seller who is somehow informed of the signal of one of two bidders and can publicly announce it. According to the above result, the public announcement is not necessarily revenue-enhancing.

This paper is organized as follows. Section 2 introduces the model. Section 3 deals with the case of IID signals while Section 4 deals with the case of affiliated signals. Section 5 concludes.

# 2 Model

A seller has an object for sale via a first-price auction, and there are two bidders, bidder 1 and 2, who commonly assess the object at $v(s)$ given signals $s = (s_1, s_2)$. Assume that

$v(s)$ is increasing in each component and symmetric in the sense that $v(s_1, s_2) = v(s_2, s_1)$. We say that $v$ is *supermodular* if, for all $s_1' > s_1$, $v(s_1', \cdot) - v(s_1, \cdot)$ is increasing, and it is *submodular* if, for all $s_1' > s_1$, $v(s_1', \cdot) - v(s_1, \cdot)$ is decreasing. Say that $v(\cdot)$ is *modular* if $v(s_1', \cdot) - v(s_1, \cdot)$ is constant for all $s_1$ and $s_1'$. If $v(\cdot)$ is differentiable, then it will be supermodular (resp. submodular) if $\frac{\partial v(s_1, \cdot)}{\partial s_1}$ is increasing (resp. decreasing) for all $s_1$.

While the distribution of signals will be specified later in each section, I consider two information structures, denoted $I^1$ and $I^2$: Bidder 1 observes $s_1$ in both $I^1$ and $I^2$, and bidder 2 observes $s_2$ only in $I^1$ but both $s_1$ and $s_2$ in $I^2$. It is assumed that the information structure is commonly known to the bidders. Note that $I^1$ corresponds to the standard setup of Milgrom and Weber (1982) and that, going from $I^1$ to $I^2$, only bidder 2's information improves. This improvement can be explained by bidder 2's activity to learn his rival's information or alternatively by the public announcement from the seller who is somehow informed of bidder 1's signal. Note also that unlike Milgrom and Weber (1982) who consider the change of information that *equally* affects all bidders, the change in information in this model affects only one bidder.

# 3 The Case of IID and Continuous Signals

Suppose that $s_1$ and $s_2$ are independently and identically distributed in the interval $[0, 1]$ according to a continuous distribution $F(\cdot)$ with density $f(\cdot)$. Assume that $v(\cdot)$ is differentiable.

In $I^1$, one can apply Milgrom and Weber (1982a) to obtain the following symmetric equilibrium bidding strategy:

$$b^1(s_i) := \int_0^{s_i} v(t, t) \left( \frac{f(t)}{F(s_i)} \right) dt.$$

In $I^2$, the equilibrium bidding strategy can be obtained by applying Engelbrecht-Wiggans, Milgrom, and Weber (1983). First, bidder 2 of type $(s_1, s_2)$ bids

$$\begin{aligned} b_2^2(s_1, s_2) &:= \mathbb{E}[v(s_1, \tilde{s}_2) | \tilde{s}_2 \leq s_2] \\ &= \int_0^{s_2} v(s_1, t) \left( \frac{f(t)}{F(s_2)} \right) dt. \end{aligned}$$

Then, bidder 1 of type $s_1$ randomizes his bid with the distribution given by

$$H_1^2(b|s_1) := \text{Prob}[b_2^2(s_1, \tilde{s}_2) \leq b] \\ = \begin{cases} 0 & \text{if } b < b_2^2(s_1, 0) \\ F(s_2) & \text{if } b = b_2^2(s_1, s_2) \text{ for some } s_2 \in [0,1] \\ 1 & \text{if } b > b_2^2(s_1, 1) \end{cases} .$$

PROPOSITION 1. *The strategy profiles* $(b^1, b^1)$ *and* $(H_1^2, b_2^2)$ *constitute a (unique) Bayesian Nash equilibrium in the first-price auction in the information structure* $I^1$ *and* $I^2$*, respectively.*

*Proof.* The proof follows from Milgrom and Weber (1982a) and Engelbrecht-Wiggans, Milgrom, and Weber (1983) and is thus omitted. ■

Using the above equilibria, we can compare bidder 2's equilibrium payoffs in two information structures as follows:

PROPOSITION 2. *Bidder 2's equilibrium payoff is higher (resp. lower) in* $I^2$ *than in* $I^1$ *if* $v(\cdot)$ *is supermodular (resp. submodular). If* $v(\cdot)$ *is modular, then bidder 2 is indifferent between* $I^1$ *and* $I^2$*.*

*Proof.* Let $u_2^k(s_2'; s_2)$ denote 2's interim payoff when he observes $s_2$ but pretends to observe $s_2'$ in each information structure $I^k, k = 1, 2$: Given the above equilibria,

$$u_2^1(s_2'; s_2) = \int_0^{s_2'} (v(s_1, s_2) - b^1(s_2'))f(s_1)ds_1$$

and

$$u_2^2(s_2'; s_2) = \int_0^1 \left(v(s_1, s_2) - b_2^2(s_1, s_2')\right) F(s_2')f(s_1)ds_1, \tag{1}$$

where the expression in (1) follows from the fact that given the equilibrium strategy profile $(H_1^2, b_2^2)$, the probability of bidder 2 winning with the bid $b_2^2(s_1, s_2')$ is equal to $F(s_2')$ irrespective of $s_1$. Let $U_2^k(s_2) := u_2^k(s_2; s_2)$, i.e. bidder 2's equilibrium interim payoff in $I^k$. Clearly, $U_2^1(0) = U_2^2(0) = 0$. To see how the equilibrium payoffs for higher types in two information structures are compared, we apply the envelope theorem

$$\frac{dU_2^k(s_2)}{ds_2} = \left.\frac{\partial u_2^k(s_2'; s_2)}{\partial s_2}\right|_{s_2'=s_2}$$

to obtain

$$\frac{d}{ds_2}[U_2^2(s_2) - U_2^1(s_2)] = \int_0^1 \frac{\partial v(s_1,s_2)}{\partial s_2} F(s_2) f(s_1) ds_1 - \int_0^{s_2} \frac{\partial v(s_1,s_2)}{\partial s_2} f(s_1) ds_1 \tag{2}$$
$$= F(s_2)\left(\mathbb{E}_{s_1}[\frac{\partial v(s_1,s_2)}{\partial s_2}] - \mathbb{E}_{s_1}[\frac{\partial v(s_1,s_2)}{\partial s_2}|s_1 \leq s_2]\right),$$

which is greater (resp. smaller) than zero if $\frac{\partial v(s_1,s_2)}{\partial s_2}$ increases (resp. decreases) in $s_1$. Given $U_2^2(0) = U_2^1(0) = 0$, this implies that $U_2^2(s_2) - U_2^1(s_2) > (<) \, 0$ for all $s_2 > 0$ if $v(\cdot)$ is supermodular (resp. submodular). Also, $U_2^2(s_2) - U_2^1(s_2) = 0$ for all $s_2$ if $v(\cdot)$ is modular. ■

To explain this result (and other results in the paper), it is important to notice that the randomized bidding strategy of bidder 1 in $I^2$ makes bidder 2's ex-post winning probability 'leveled out' as the information structure changes from $I^1$ to $I^2$. That is, for any given signal $s_2$, bidder 2 is winning more (resp. less) often against high (resp. low) rival types in $I^2$ than in $I^1$, though the interim winning probability remains the same in both information structures.[2] The impact of this leveling-out on bidder 2's payoff depends on whether the value is supermodular or submodular. To see it, consider the marginal change in bidder 2's interim payoff as $s_2$ increases, which consists of two parts: the direct part due to the marginal increase in value and the indirect part due to the marginal increase in bidder 2's winning probability. While only the direct part survives according to the envelope theorem, its magnitude, as shown in (2), depends on what rival types bidder 2 is winning against in each information structure. Given the leveling-out of ex-post winning probability in $I^2$, he is more likely to win against low types of bidder 1 in $I^1$ than in $I^2$, which implies that if the value is submodular, then the value-enhancing effect of a higher $s_2$, i.e. the direct part, will be greater in $I^1$ than in $I^2$ while the opposite will hold if the value is supermodular. This, combined with the fact that the lowest type obtains zero payoff in both information structures, yields the result in Proposition 2.

A corollary of Proposition 2 is that if the value is submodular or modular, then the seller's revenue is higher in $I^2$, which is immediate from the fact that going from $I^1$ to $I^2$, bidder 2's payoff does not increase and bidder 1's payoff decreases to zero while the total surplus generated remains the same.

[2]This can be seen from the fact that given any $s_2$, in $I^1$, bidder 2 wins (resp. loses) for sure if $s_1$ is lower (resp. higher) than $s_2$ while, in $I^2$, he wins with the same probability $F(s_2)$ irrespective of $s_1$

Corollary 1. *The seller's revenue is higher in $I^2$ than in $I^1$ if $v(\cdot)$ is submodular or modular.*

If the value is supermodular in the signals, bidder 2's payoff rises and bidder 1's payoff falls to zero as the information structure changes from $I^1$ to $I^2$, which makes it unclear whether the seller's revenue increases or not. The following example presents a case of supermodular value in which bidder 2's payoff becomes so large as to make the seller's revenue lower in $I^2$.

Example 1. *Suppose that $v(s_1, s_2) = (s_1+s_2)^k$ where $k > 1$ so $v(\cdot)$ is supermodular, and both $s_1$ and $s_2$ are uniformly distributed on $[0, 1]$. The seller's revenue is $R^1 = \frac{2^{k+1}}{(k+1)(k+2)}$ in $I^1$ while it is $R^2 = \frac{2^{k+3}-k-5}{(k+1)(k+2)(k+3)}$ in $I^2$. Thus, $R^1 > R^2$ if $k > \overline{k} \simeq 4.605$, so the revenue is lower in $I^2$.*

Comparing the *interim* bidding schedules in two information structures will be helpful for understanding the role of supermodularity in the revenue ranking of the above example.[3] To this end, we focus on bidder 2's interim bidding schedule since two bidders adopt a symmetric bidding *distribution* in both information structures. First, observing low signals, bidder 2 bids more aggressively in $I^2$: For instance, with $s_2 = 0$, he bids $v(0, 0)$ in $I^1$ while bidding on average $\mathbb{E}_{s_1}[b_2^2(s_1, 0)] = \mathbb{E}_{s_1}[v(s_1, 0)] > v(0, 0)$ in $I^2$. This may put an upward pressure on higher types' bids and thus shift up the entire bidding schedule in $I^2$, which will translate into the revenue ranking as in the Corollary 1. With supermodular values, however, the interim bidding schedule tends to increase faster in $I^1$ than in $I^2$ as $s_2$ increases, which may reverse the ranking as in the Example 1. The reason is again related to the leveling-out of bidder 2's ex-post winning probability in $I^2$, which implies that as bidder 2 increases the bid, his winning probability increases against *all rival types* in $I^2$ while it increases against *marginal rival types* in $I^1$. Thus, for high values of $s_2$, the marginal benefit of raising his bid tends to be greater in $I^1$ than in $I^2$ if the value is supermodular so that bidder 2 has better incentive to raise his bid in $I^2$ for high $s_2$'s. This pattern is depicted in the Figure 1 below where $B^i$ corresponds to bidder 2's interim bidding schedule in $I^i$ for the value function given in the Example 1 with $k = 5$.

[3]The interim bidding schedule for bidder 2 refers to the expected amount of bids by bidder 2 conditioinal on the realization of $s_2$, which will be $b^1(s_2)$ in $I^1$ and $\mathbb{E}_{s_1}[b_2^2(s_1, s_2)]$ in $I^2$.

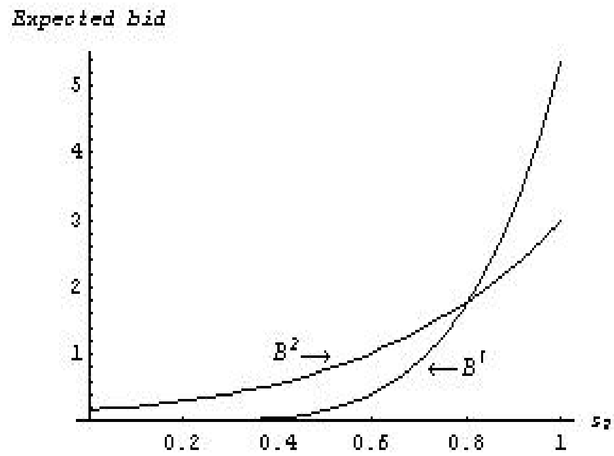


Figure 1: Comparison of bidder 2's interim bidding schedules

Remark 1. *Recalling the OCS auction example from the Introduction, suppose that prior to the auction stage, bidder 2 can act to acquire signals at a negligible cost, which is observable to bidder 1. Assuming that $s_1$ is already known to bidder 1, bidder 2 has two options: Acquire $s_2$ only or both $s_1$ and $s_2$. Proposition 2 tells that if the value is submodular, then bidder 2 will choose to acquire $s_2$ only. So, the revenue increase predicted in Corollary 1 will not be realized. In contrast, if the value is supermodular, then bidder 2 will choose to acquire both signals, which may reduce the seller's revenue as in Example 1. This observation suggests that if the decision of information acquisition is endogenized, the increase in seller's revenue from one bidder's learning of his rival's information may be less pronounced than expected.*

# 4 The Case of Affiliated and Binary Signals

The findings in Proposition 2 and Example 1 seem to be in contrast with the result of CL in which both seller's revenue and informed bidder's payoff are higher in $I^2$ with affiliated (and binary) signals. This result, however, is limited due to the restriction that the value is modular in the signals. It is shown in this section that if the value is allowed to be sub/supermodular, then the results similar to those in the IID case still hold with affiliated signals.

Following CL, I assume that the underlying value of the object, $v$, is equally likely to be 0 or 1. Signals $s_1$ and $s_2$ are assumed to be binary and IID conditional on a realized value $v$: For each $i = 1, 2$,

$$\alpha_0 := \text{Prob}[s_i = 0 | v = 0] \quad \text{and} \quad \alpha_1 := \text{Prob}[s_i = 1 | v = 1]$$

with $\alpha_0, \alpha_1 \geq \frac{1}{2}$ for affiliation. Let $p(s_1, s_2)$ denote the probability that $(s_1, s_2)$ is realized. Letting $v(s_1, s_2)$ denote the expected value of the object conditional on $(s_1, s_2)$ being realized, we have

$$v(0,0) = \frac{(1-\alpha_1)^2}{\alpha_0^2 + (1-\alpha_1)^2}, \; v(1,1) = \frac{\alpha_1^2}{\alpha_1^2 + (1-\alpha_0)^2}$$

and

$$v(1,0) = v(0,1) = \frac{\alpha_1(1-\alpha_1)}{\alpha_0(1-\alpha_0) + \alpha_1(1-\alpha_1)}.$$

Also, letting $v(s_i)$ denote the expected value of the object conditional on $s_i$ being realized, we have

$$v(0) = \frac{1-\alpha_1}{1-\alpha_1+\alpha_0} \text{ and } v(1) = \frac{\alpha_1}{1-\alpha_0+\alpha_1}.$$

It is important to notice that the value is supermodular if $(v(1,1)-v(0,1))-(v(1,0)-v(0,0)) > 0$ or $v(1,1)+v(0,0)-2v(1,0) > 0$, which holds if and only if

$$\alpha_0(1-\alpha_0) - \alpha_1(1-\alpha_1) > 0,$$

or $1/2 \leq \alpha_0 < \alpha_1$. Likewise, the value is submodular if $1/2 \leq \alpha_1 < \alpha_0$. CL assume that $\alpha_0 = \alpha_1 = \alpha \in [1/2, 1]$, i.e. the value is modular. One can also verify that when $\alpha_1 > \alpha_0$, $\frac{\partial}{\partial \alpha_1}(v(1,1)+v(0,0)-2v(1,0)) > 0$, implying the degree of supermodularity increases as $\alpha_1$ gets larger with $\alpha_0$ fixed, and that when $\alpha_1 < \alpha_0$, $\frac{\partial}{\partial \alpha_0}(v(1,1)+v(0,0)-2v(1,0)) < 0$, implying the degree of submodularity increases as $a_0$ gets larger with $\alpha_1$ fixed.

As in the previous section, I derive the equilibrium bidding strategy for each information structure and compare the bidders' equilibrium payoffs and the seller's revenues across the two information structures. I closely follow CL for the derivation of equilibrium strategies.

• **Information structure $I^1$.** The equilibrium bidding strategy is symmetric: If $s_i = 0$, each bidder $i$ bids $v(0,0)$ with probability one while if $s_i = 1$, he randomizes in the interval $[v(0,0), \frac{\alpha_0^2(2\alpha_1-1)+(1-\alpha_1)^2(\alpha_1+\alpha_0)}{(\alpha_0^2+(1-\alpha_1)^2)(1+\alpha_1-\alpha_0)}]$ with the distribution given by

$$H^1(b) := \left(\frac{\alpha_0(1-\alpha_0)+\alpha_1(1-\alpha_1)}{\alpha_0^2+(1-\alpha_1)^2}\right)\frac{(\alpha_0^2+(1-\alpha_1)^2)x-(1-\alpha_1)^2}{\alpha_1^2-(\alpha_1^2+(1-\alpha_0)^2)x}.$$

As a result, two bidders obtain a symmetric ex ante payoff

$$\pi^1 := p(1,0)\,(v(1,0) - v(0,0)) \tag{3}$$

and the seller's revenue is $R^1 := 0.5 - 2\pi^1$.

• **Information structure $I^2$.** If $s_1 = 0$, bidder 1 randomizes in the interval $[v(0,0), v(0)]$ with the distribution given by

$$H_1^2(x|0) := \frac{\alpha_0(1-\alpha_1)(\alpha_0+\alpha_1-1)}{(1-\alpha_0+\alpha_1)[\alpha_1(1-\alpha_1) - x(\alpha_0(1-\alpha_0)+\alpha_1(1-\alpha_1))]}.$$

If $(s_1, s_2) = (0,0)$, bidder 2 bids $v(0,0)$ with probability one while if $(s_1, s_2) = (0,1)$, he randomizes in the interval $[v(0,0), v(0)]$ with the distribution given by

$$H_2^2(x|0,1) := \frac{(\alpha_0^2 + (1-\alpha_1)^2)x - (1-\alpha_1)^2}{\alpha_1(1-\alpha_1) - x(\alpha_0(1-\alpha_0)+\alpha_1(1-\alpha_1))}.$$

If $s_1 = 1$, bidder 1 randomizes in the interval $[v(1,0), v(1)]$ with the distribution given by

$$H_1^2(x|1) := \frac{\alpha_1(1-\alpha_0)(\alpha_0+\alpha_1-1)}{(1-\alpha_0+\alpha_1)(\alpha_1^2 - x(\alpha_1^2+(1-\alpha_0)^2))}.$$

If $(s_1, s_2) = (1,0)$, bidder 2 bids $v(1,0)$ with probability one while if $(s_1, s_2) = (1,1)$, he randomizes in the interval $[v(1,0), v(1)]$ with the distribution given by

$$H_2^2(x|1,1) := \frac{x(\alpha_0(1-\alpha_0)+\alpha_1(1-\alpha_1)) - \alpha_1(1-\alpha_1)}{\alpha_1^2 - x(\alpha_1^2 + (1-\alpha_0^2))}.$$

As a result, the equilibrium payoffs of bidder 1 and 2 are

$$\begin{aligned}\pi_1^2 &:= 0\\ \pi_2^2 &:= p(0,1)\,(v(0,1) - v(0)) + p(1,1)\,(v(1,1) - v(1)),\end{aligned}$$

respectively, and thus the seller's revenue is $R^2 := 0.5 - \pi_2^2$.

The contour plots in Figure 2 and 3 depict the difference in bidder 2's payoffs and the difference in seller's revenues across $I^1$ and $I^2$, respectively. Noting that a whiter color represents a greater magnitude, we observe that the payoff difference, $\pi_2^2 - \pi^1$, tends to be greater as the value is more supermodular (or $\alpha_1$ is large relative to $\alpha_0$) while the revenue difference, $R^2 - R^1$, tends to be greater as the value is more submodular (or $\alpha_0$ is large relative to $\alpha_1$).

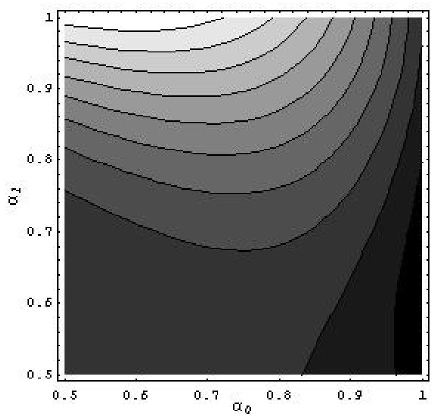

Figure 2: Contour sets for $\pi_2^2 - \pi^1$

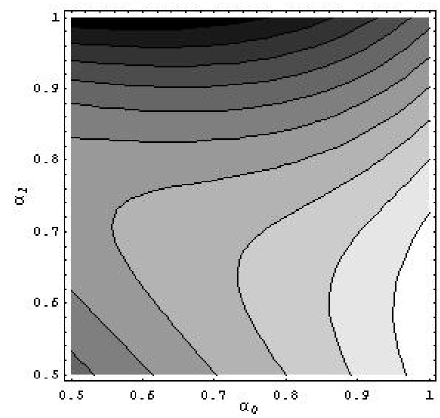

Figure 3: Contour sets for $R^2 - R^1$

Also, Figure 4 below shows that bidder 2 can be worse off in $I^2$ if the value is highly submodular, which corresponds to the area $B$. As for the seller's revenue, Figure 5 shows that it can be lower in $I^2$ if signals are highly supermodular, which corresponds to the area $C$. These findings are consistent with those obtained in the previous section.

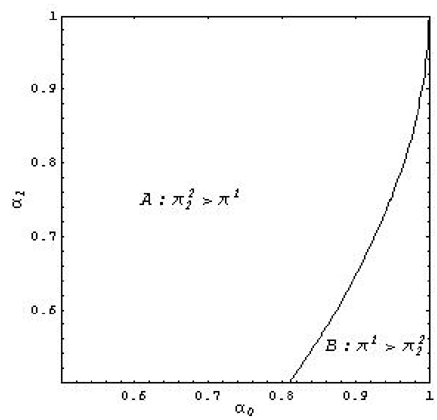


Figure 4: Comparison of bidder 2's payoffs

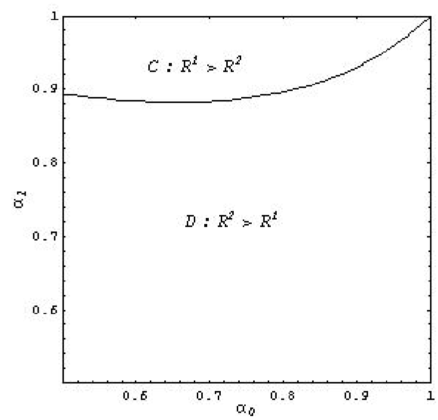


Figure 5: Comparison of seller's revenues

Note that the comparison of bidder 2's payoffs is not as clear-cut as in Proposition 2 since bidder 2 is better off in $I^2$ when $\alpha_1 = \alpha_0$, i.e. value is modular as in CL. To explain this difference, note that in the IID case, bidder 2's ex-post winning probability gets *completely* leveled out going from $I^1$ to $I^2$, meaning that it remains the same irrespective of $s_1$ as long as $s_2$ is fixed. With affiliated signals, however, it can be seen that the winning probability in $I^2$ is still higher against bidder 1 with low signal than with high signal. Due to this, going from $I^1$ to $I^2$, the bid of bidder 2 with low signal will not increase as much as in the IID case, which will put less pressure on high type's bids also. For the same reason, the effect of the sub/supermodularity on the slope of equilibrium bidding schedules will not be as pronounced as in the IID case. Considering these, we may not expect the comparison to be as sharp as in Proposition 2. However, as shown in the above figures, the qualitative features of the results are very similar to those in the IID case.

Remark 2. *The above result shows that the seller's revenue can be non-monotonic with respect to bidders' information.*[4] *First, as only one out of two bidders learns his rival's signal, the seller's revenue falls if the value is sufficiently supermodular. As another bidder also learns his rival's signal, it rises back since bidders compete with complete information so the seller can extract the entire surplus.*

# 5 Concluding Remarks

Since Milgrom and Weber (1982a) established the linkage principle, there have been studies showing that the public announcement that equally improves all bidders' information is not necessarily revenue-enhancing: For instance, Perry and Reny (1999) where bidders are multi-units demanders, Fang and Parreiras (2003) where bidders are budget constrained, and Feinberg and Tennenholtz (2005) where bidders cannot observe the identity of bidders who have dropped in an English auction. The current paper can be viewed as complementing the above literature by showing that an asymmetric improvement of bidders' information need not be revenue-enhancing, either. What is rather surprising is that one bidder, by learning his rival's information, can reduce his own payoff as well as the seller's revenue. This, of course, is because different information status of one bidder induces different response from his rival. The improved information of one bidder can sometimes induce such an aggressive response from the rival as to make the former's payoff decrease.

[4] A similar type of non-monotonicity has also been reported in Kim and Che (2004) in the IPV setup.

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