

# Screening Loss Averse Consumers\*

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## Abstract

Sellers often discriminate heterogeneous consumers with just a few products. This paper proposes an explanation for such coarse screening, based on consumer loss aversion. In our model, a seller offers a menu of bundles before a consumer learns his willingness to pay, and the consumer experiences gain-loss utility with reference to his prior (rational) expectation *à la* Köszegi and Rabin (2006). For the case of binary consumer types, we show that the seller finds it optimal to offer a *pooling* menu under an intermediate range of loss aversion if the likelihood of low willingness-to-pay consumer is sufficiently large. We also identify sufficient conditions under which partial or full pooling dominates screening for the case of continuous types.

**JEL Classification:** D03, D82, D86

**Keywords:** Reference-dependent preferences, loss aversion, price discrimination, screening menu, pooling menu

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# 1 Introduction

When facing heterogeneous buyers, price discrimination allows a seller to capture a larger portion of the total market surplus than offering a single product quality. Price discrimination is indeed prevalent but sellers often employ just a small number of product types, despite our casual and statistical observations that suggest significant heterogeneity among consumers' willingness to pay.<sup>1</sup> Such *coarse* or simple screening has been commonly attributed to the existence of some fixed costs of launching products of different qualities. In many instances (e.g. cable TV packages or software versions), however, these costs tend to be small so it is difficult to justify the relative lack of quality diversity by resorting to such costs alone.

This paper aims to offer another explanation for coarse screening, based on consumer loss aversion, by introducing Kőszegi and Rabin's (2006) expectation model of reference-dependent preferences into a standard screening model *à la* Mussa and Rosen (1978).<sup>2</sup> Our motivation for this approach to the question of coarse screening is two-fold.<sup>3</sup> First, in addition to the large existing literature documenting empirical support for loss aversion in a variety of economic situations, several recent studies point to the specific role played by expectations in formation of reference points (Mas (2006), Crawford and Meng (2009) and Abeler, Falk, Götte and Huffman (2009)).<sup>4</sup> Second, in many applications of price discrimination, from ticketing to cable TV plans, a time lag exists between menu offer and consumption opportunity such that (i) when the seller offers a menu, the buyer faces uncertainty over his willingness to pay and, hence, (ii) when the buyer learns his type and makes a consumption

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<sup>1</sup>Crawford and Shum (2007) find that the US cable television industry largely consists of firms offering a single product quality. Leslie (2004) estimates many different consumer valuations for a Broadway play but observes that the majority of performances come with just two seat qualities.

<sup>2</sup>Kőszegi and Rabin (2007, 2009) extend their previous model to incorporate risky and intertemporal decisions. Other models of expectation-based reference-dependent preferences are analyzed by Bell (1985), Loomes and Sugden (1986), Gul (1991) and Shalev (2000).

<sup>3</sup>See Ellison (2006) for a broad perspective on studying firm behavior under less than fully rational consumers.

<sup>4</sup>Evidence suggest that loss aversion can account for diverse economic phenomena, from the equity premium puzzle (Benartzi and Thaler, 1995) to the seller behavior in Boston housing market (Genesove and Mayer, 2001). See Camerer (2006) for a survey.

choice, he may compare his choice to what he could have consumed.

Often a buyer's willingness to pay for a good or service depends on the realization of a specific condition, such as the level of disposable income, and it is plausible to think that such a buyer formulates some contingent consumption plan *ex ante*. Consider, for example, a buyer who is planning to begin or renew a cable TV subscription for the holiday season. The buyer's preferences over various packages of channels (e.g. economy or premium package) can depend on the amount of bonus that he will receive.<sup>5</sup> The buyer thus expects that if the bonus is generous, he becomes more willing to pay for the premium package relative to the economy one while otherwise his incremental valuation for the premium package is low.

We consider a setup in which a seller offers a menu of bundles before a buyer privately observes his willingness to pay and decides whether or not to make a purchase. As in Kőszegi and Rabin (2006), henceforth referred to as KR, the buyer anticipates his future consumption choice for each possible contingency and experiences "gain-loss utility" with reference to his own past expectation of contingent consumption, in addition to standard "consumption/intrinsic utility." Furthermore, the expectation must be correct; that is, it must be consistent with the buyer's optimal consumption choice in each realization of uncertainty. This requirement of rational expectation, or *personal equilibrium*, implies that the menu must satisfy incentive compatibility and (ex-post) participation constraints that account for reference-dependent preferences and loss aversion.

Our main message is most clearly conveyed in the case of binary consumer types. Here, the effect of loss aversion manifests itself in two ways. First, when the buyer consumes a bundle ex post, he compares it with the other bundle, that is, an incentive compatibility constraint must be met. But, in our setup this requirement is complicated by the presence of gain-loss utility. For instance, a deviation to lower quality-price bundle entails a loss in terms of reduced quality but a gain of lower price paid. Furthermore, each comparison is weighted by the likelihood of the alternative outcome. Thus, the incentive compatibility

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<sup>5</sup>It is well known that differences in consumer preferences can be generated from differences in income (see Tirole (1988, p.96 and p.143) and Shaked and Sutton (1982)). If all consumers have identical preferences for the good but different incomes, they exhibit different marginal rates of substitution between income and quality. Then, the consumer type can be interpreted as the inverse of the marginal rate of substitution, as in this cable TV example.

constraint responds ambiguously to an increase in the loss aversion parameter, depending on the distribution of consumer types. Second, the consumer compares the bundle of his choice with the alternative of non-participation. Compared to each bundle, non-participation delivers a loss on quality and a gain on price. Thus, as the buyer becomes more loss averse, this has an effect of making the consumer willing to pay more for a given quality, which implies that the seller can profitably increase the quality for the type whose participation constraint is binding (i.e. the low type).

The combination of the above two effects generates a very different characterization of the seller's optimal menu in our setup, where different types of buyers with standard preferences would be separated via a menu with strictly increasing quality/payment schedule. Our main result establishes the following: when the likelihood of low willingness-to-pay consumer is sufficiently large and the degree of loss aversion lies in an intermediate range, the seller's optimal menu is to offer the same bundle to both types, i.e. a *pooling* menu.

In the absence of asymmetric information, however, pooling does not arise in optimum; instead, the seller finds it optimal to offer the loss averse consumer a menu that endows the low type with a higher quality but charges the high type a larger price.<sup>6</sup> Clearly, such a menu is not incentive compatible with asymmetric information, and this drives the optimality of pooling when loss aversion is not severe.<sup>7</sup> This demonstrates that the *interplay* between loss aversion *and* informational asymmetry generates the optimality of pooling.

We extend our analysis to the case of continuous types. In our setup this case presents a formidable challenge for computing the optimal menu because loss aversion renders a huge variety of contractual forms as potential solution candidates. Nonetheless, the following result is obtained for the case in which full separation is optimal under standard preferences: under a certain distributional assumption, and when the buyer is sufficiently loss averse, partial or even full pooling dominates screening.

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<sup>6</sup>This comes from the fact that even when consuming a lower quality the high type enjoys a higher utility from quality consumption and thus is charged a larger price under the optimal menu with symmetric information. The details are given in Section 4.1.

<sup>7</sup>When the buyer is sufficiently loss averse, a reverse-screening menu, where the low consumer type purchases a higher quality-price bundle than the high consumer type, can be made incentive feasible and even optimal.

These results, above all, contribute to the literature that has long been concerned with the question of why we often observe *simple* contracts (see, for instance, Holmstrom and Milgrom (1987) for an early account). Recently, Herweg, Müller and Weinschenk (2009) adopt the reference-dependent preference framework of KR in the incentive design problem with moral hazard. In a similar vein to our predictions, they show that the optimal contract involves only binary payment schemes. In an alternative effort provision setup, Hart and Moore (2008) also derive the role of reference dependence in generating simple optimal contracts.

Our paper also adds to the growing literature on firm behavior under biased consumer preferences (see Armstrong and Huck (2010) for a recent survey). Within this literature, we are most closely related to Heidhues and Köszegi (2005, 2008) who show that consumer loss aversion in the framework of KR’s expectation-based reference point can explain why a monopolist may adopt a sticky pricing schedule and firms with differentiated products and heterogeneous costs may end up charging a uniform price. Pricing for consumers with time-inconsistent preferences or self-control problems has been analyzed by, among others, DellaVigna and Malmendier (2004), Eliaz and Spiegel (2006), Esteban, Miyagawa and Shum (2007) and Heidhues and Köszegi (2009). The role of overconfident consumers has been explored by Eliaz and Spiegel (2008) and Grubb (2009).

This paper is organized as follows. Section 2 lays out a price discrimination setup with KR’s reference-dependent preferences for the binary type case, followed by a characterization of the optimal menu in Section 3. In Section 4, we discuss how our main result, i.e. the optimality of pooling, is affected by the lack of informational asymmetry or the consideration of ex-ante participation. We extend the analysis to the continuous type case in Section 5. Some concluding comments are offered in Section 6. All proofs are relegated to the Appendix unless mentioned otherwise. We also present the details of some omitted results and proofs in a Supplementary Material.

## 2 The Setup

Consider a market that consists of a monopolistic seller of some product and its buyer. Let  $b = (q, t)$  denote a ‘bundle’ in which the product of quality  $q$  is sold for  $t$  dollars of payment,

and let  $\emptyset = (0, 0)$  denote the null bundle. The seller’s profit from a bundle  $b = (q, t)$  is  $t - cq$ , where  $c > 0$  is the constant marginal cost of production.

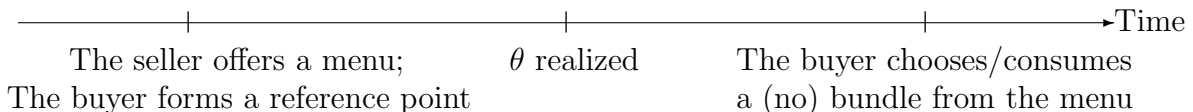
We assume that the buyer has reference-dependent preferences, which consist of “consumption” (or intrinsic) utility and “gain-loss” utility. First, the buyer’s consumption utility from a bundle  $b = (q, t)$  is given by

$$m(b; \theta) \equiv \theta v(q) - t,$$

where  $\theta$  is a measure of willingness-to-pay and  $v(\cdot)$  is a (differentiable) function satisfying  $v(0) = 0$ ,  $v'(\cdot) > 0$ ,  $v''(\cdot) < 0$ ,  $\lim_{q \rightarrow 0} v'(q) = \infty$ , and  $\lim_{q \rightarrow \infty} v'(q) = 0$ . We assume that  $\theta$  is randomly drawn and becomes privately known to the buyer when it is realized. For now,  $\theta$  is assumed to take binary values,  $\theta_L > 0$  and  $\theta_H > \theta_L$  with probability  $p \in (0, 1)$  and  $1 - p$ , respectively. Later in Section 5, we will consider a continuum of types.

Next, we explain the gain-loss utility. The situation we have in mind is as follows. The seller commits to a menu of bundles,  $M$ , which is observed by the buyer *before* his willingness-to-pay,  $\theta$ , is drawn. The buyer then anticipates his choice of bundle for each possible contingency, and this expectation serves as his reference point when actual consumption takes place *after* the uncertainty gets resolved. We will later require that this expectation should be consistent with the buyer’s actual choices, that is, the expectation should be *rational*.

The following timeline will be useful to illustrate the model and compare it with the standard screening model.



In the standard contract theory,  $\theta$  is casually assumed to represent the agent’s inherent preference for consumption which is realized and privately observed prior to the contracting stage. In our setup,  $\theta$  can be interpreted as a random utility component that becomes known to the buyer only when the uncertainty, which determines the buyer’s willingness to pay, gets resolved around the time of consumption (e.g. the amount of bonus in the example mentioned in Introduction above). Prior to learning  $\theta$  and making actual purchase

decision, the buyer has time to observe the seller’s menu already present in the market (and possibly being bought by other buyers) and forms a contingent consumption plan that will serve as reference point.<sup>8</sup> We see the above timeline as being consistent with many real world situations. Its implication for our analysis is that any feasible menu should satisfy the *ex-post* participation constraint, as will become clear shortly.<sup>9</sup>

Formally, let  $r_i = (q_i^r, t_i^r)$  denote the bundle that the buyer expects to choose if his type is realized to be  $\theta_i$ ,  $i = H, L$ . Let  $R \equiv \{r_L, r_H\}$ . Then, the gain-loss utility of buyer type  $\theta$  from choosing a bundle  $b = (q, t)$  takes the following form:

$$n(b; \theta, R) \equiv p [\mu(\theta v(q) - \theta_L v(q_L^r)) + \mu(t_L^r - t)] + (1 - p) [\mu(\theta v(q) - \theta_H v(q_H^r)) + \mu(t_H^r - t)], \quad (1)$$

where  $\mu$  is an indicator function such that, for any  $k_1, k_2 \in \mathbb{R}_+$ ,

$$\mu(k_1 - k_2) \equiv \begin{cases} k_1 - k_2 & \text{if } k_1 \geq k_2 \\ \lambda(k_1 - k_2), \lambda > 1 & \text{if } k_1 < k_2. \end{cases}$$

The term  $\mu(\theta v(q) - \theta_L v(q_L^r))$  in the RHS of (1), for instance, captures type  $\theta$ ’s gain/loss from consuming quality  $q$  relative to  $q_L^r$ , the level that he *would* have consumed had the realization of uncertainty been  $\theta_L$ , which is then weighted by probability  $p$  with which the buyer had expected  $\theta_L$  to occur. Other terms can be explained similarly. Here the parameter  $\lambda$  measures the degree of loss aversion;  $\lambda > 1$  means that the buyer is *loss-averse*, i.e. more sensitive to loss than to gain.

Note that the gain-loss utility is additively separable across the two consumption dimensions, quality and transfer, following Tversky and Kahneman (1991) and Koszegi and Rabin (2006). This is a key feature of the reference-dependent preference theory that successfully explains the *endowment effect* (e.g. the gap between sellers’ willingness to accept and buyers’

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<sup>8</sup>Note here that we are restricting the buyer’s reference point to come only from bundles that are offered by the seller and thus actually consumable. This is in line with KR’s approach of endogenizing reference point as rational expectation about future contingent behavior. One could also justify this restriction by appealing to the buyer’s effort to search for the existing product bundles and/or the seller’s marketing activities to promote her own product bundles.

<sup>9</sup>We also explore the robustness of our results to adding an ex-ante participation constraint. See Section 4.2.

willingness to pay for the same object), which has regularly been observed in a number of empirical and experimental studies but cannot be explained by the standard theory.<sup>10</sup>

Given the expected choices of bundles, or reference point,  $R$ , the overall utility of buyer type  $\theta$  from  $b = (q, t)$  is the sum of consumption and gain-loss utilities:<sup>11</sup>

$$u(b|\theta, R) := m(b; \theta) + n(b; \theta, R).$$

We now introduce to our setup the notion of *personal equilibrium* proposed by KR, which incorporates the idea that the reference point formed by an economic agent should be in accordance with his actual choices:

**Definition 1.** *Given any menu  $M$ ,  $R = \{r_i\}_{i=H,L} \subseteq M$  is a personal equilibrium (PE) if*

$$u(r_i|\theta_i, R) \geq u(b|\theta_i, R), \forall b \in M \cup \{\emptyset\}, \forall i = H, L. \quad (2)$$

*Furthermore,  $R = \{r_i\}_{i=L,H}$  is a truthful personal equilibrium (TPE) if it is a PE given  $M = R$ .*

Condition (2) requires that each bundle  $r_i$  in the PE is optimal for type  $\theta_i$  with  $R$  as the reference point so that  $r_i$  is the bundle the buyer actually chooses if his type turns out to be  $\theta_i$ . Note that the equilibrium utility of each type must be no lower than its utility from choosing null option since the buyer can always opt out after realizing his type, consistent with our timeline given above.

In the case of a TPE, the reference point itself is offered as a menu and therefore each type only needs to prefer his choice of bundle over the other type's bundle or the null bundle. That is,  $R = \{r_i\}_{i=L,H}$  is a TPE if and only if the incentive compatibility and individual rationality requirements hold as follows: for each  $i = H, L$ ,

$$u(r_i|\theta_i, R) \geq u(r_{-i}|\theta_i, R) \quad (IC_i)$$

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<sup>10</sup>An alternative reference formation is to calculate the gain-loss utility in terms of the total consumption utility,  $\theta v(q) - t$ . Such a model, however, does no better at explaining the endowment effect than the standard theory. We also show that, in our setup, it produces results no different from the standard model; see the Supplementary Material for details.

<sup>11</sup>To adjust the magnitude of gain-loss utility relative to consumption utility, we could introduce a parameter, say  $\beta$ , and multiply it to the gain-loss utility term. Here, we set  $\beta$  equal to 1 for simplicity; the qualitative features of our results remain valid for any  $\beta$  if it is not too small.

$$u(r_i|\theta_i, R) \geq u(\emptyset|\theta_i, R). \quad (IR_i)$$

Since these inequalities, henceforth referred to as  $(IC)$  and  $(IR)$  constraints, are implied by (2), the following result is immediate.

**Proposition 1.** *Consider any menu  $M$ , and suppose that  $R = \{r_i\}_{i=L,H}$  is a personal equilibrium given  $M$ . Then,  $R$  is a truthful personal equilibrium.*

Given this result, there is no loss in restricting ourselves to menus that are themselves TPEs, when searching for the seller's optimal menu. We shall sometimes refer to such a menu simply as a TPE menu.

Note, however, that given a TPE menu, another PE may still exist in which the buyer is non-truthful, i.e. some type  $\theta_i$  chooses  $r_{-i}$  or  $\emptyset$ . One problem that can arise in the case of multiple personal equilibria is that the buyer may be better off with the non-truthful equilibrium than with the truthful one. In particular, the buyer might incur a negative *ex-ante* expected utility from playing a TPE, and there could be another PE in which the buyer never consumes. We show, in Section 4.2, that requiring TPE menus not to cause an *ex-ante* loss to the buyer in fact does not change, and rather strengthens, our main result that pooling often dominates screening.<sup>12</sup>

### 3 Optimal Menu with Binary Types

The seller's problem, denoted by  $[P]$ , is given by

$$\max_{\{(q_L, t_L), (q_H, t_H)\}} p(t_L - cq_L) + (1 - p)(t_H - cq_H) \quad [P]$$

subject to  $R = \{(q_L, t_L), (q_H, t_H)\}$  being a TPE, that is,  $R$  satisfying the  $(IC)$  and  $(IR)$  constraints defined above.

Under the reference-dependent preference framework, a broader class of menus can be supported as TPEs, compared to the standard screening model. In particular, it is possible

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<sup>12</sup>One way to fully address the issue of multiple personal equilibria is to characterize an optimal menu by considering menus that are not only TPEs but also maximize the buyer's payoff among all PEs (as in KR's *preferred personal equilibrium or PPE*). This is beyond the scope of this paper.

to have the low type buyer choosing a higher quality/price pair than the high type since the high type may obtain a loss from deviating to mimic the low type. Such possibilities cannot therefore be ruled out by simply relying on the usual incentive compatibility arguments. A unified analysis of all possible menus is not available, however, since different classes of menus entail different forms of gain-loss utility. Our analysis below considers each class separately to identify an optimal menu within that class, which will then lead us to characterize the overall optimal menu.

One class of menus that can be easily ruled out is the one where one type of buyer receives a lower quality but pays more than the other type does (including the case of a higher payment for the same quality or the same payment for a lower quality). The reason is simple: if the former type deviates to the latter's bundle, then he will enjoy a higher gain-loss utility as well as a higher intrinsic utility.

We are therefore left with the following three classes of menus to consider:

1. *Pooling menu*:  $q_H = q_L$  and  $t_H = t_L$
2. *Screening menu*:  $q_H > q_L$  and  $t_H > t_L$
3. *Reverse-screening menu*:  $q_H < q_L$  and  $t_H < t_L$

We let  $\mathcal{M}^P$ ,  $\mathcal{M}^S$  and  $\mathcal{M}^R$  denote the set of pooling, screening and reverse-screening menus, respectively, that satisfy the *(IC)* and *(IR)* constraints. Whenever we mention an “optimal screening” menu, for instance, it is meant to be optimal with the set of screening menus. For the full expressions of relevant *(IC)* and *(IR)* constraints, we refer the reader to the Supplementary Material.

### 3.1 Pooling Menu

We begin by characterizing the seller's profit-maximizing choice among the pooling menus. Consider a pooling menu  $R = \{r = (q, t)\} \in \mathcal{M}^P$ . Clearly, the *(IR<sub>H</sub>)* constraint is implied by the *(IR<sub>L</sub>)* constraint since, if both types choose the same bundle, type  $\theta_H$  is better off in terms of both intrinsic and gain-loss utilities while the outside payoff is type-independent.

Now,  $(IR_L)$  can be written as

$$\begin{aligned} u(r|\theta_L, R) &= \theta_L v(q) - t - (1-p)\lambda(\theta_H - \theta_L)v(q) \\ &\geq u(\emptyset|\theta_L, R) = p[t - \lambda\theta_L v(q)] + (1-p)[t - \lambda\theta_H v(q)], \end{aligned}$$

or after rearrangement,

$$t \leq \frac{(\lambda + 1)}{2} \theta_L v(q). \quad (3)$$

Clearly, (3) must be binding at the optimum. The following result is then immediate from the first-order condition of the seller's profit-maximization.

**Proposition 2.** *The optimal pooling menu,  $\{(q^p, t^p)\}$ , is such that  $\theta_L v'(q^p) = \frac{2c}{\lambda+1}$ .*

Thus, the seller finds it optimal to sell a higher quality as the buyer gets more loss-averse, i.e.  $\lambda$  gets bigger. This is because the buyer wants to avoid the loss from non-participation and, therefore, is willing to pay more for a given amount of consumption if he is more loss-averse, as can be seen in (3) above.

### 3.2 Screening Menu

Consider a screening menu  $R = \{r_L = (q_L, t_L), r_H = (q_H, t_H)\} \in \mathcal{M}^S$  where  $q_L < q_H$  and  $t_L < t_H$ . As in the standard screening model, we can show that  $(IC_H)$  and  $(IR_L)$  constraints are binding at the optimum while the other constraints are not. Using a similar derivation to (3), the binding  $(IR_L)$  constraint can be written as

$$t_L = \frac{\lambda + 1}{2} \theta_L v(q_L). \quad (4)$$

Thus, for the same reason as in the optimal pooling menu above, the optimal quality for the low type increases with loss aversion. We will refer to this as the *participation effect* of loss aversion, meaning that a greater aversion to the loss resulting from comparison with non-participation enables the seller to charge more and thus increase the quality for the low type consumer.

Next, write the  $(IC_H)$  constraint as

$$u(r_H|\theta_H, R) = \theta_H v(q_H) - t_H + p[\theta_H v(q_H) - \theta_L v(q_L) - \lambda(t_H - t_L)]$$

$$\begin{aligned} \geq u(r_L|\theta_H, R) &= \theta_H v(q_L) - t_L + p(\theta_H - \theta_L)v(q_L) \\ &\quad + (1-p)[(t_H - t_L) - \lambda\theta_H(v(q_H) - v(q_L))], \end{aligned}$$

which can then be rewritten as

$$[1 + (1-p) + p\lambda](t_H - t_L) \leq [1 + p + (1-p)\lambda]\theta_H[v(q_H) - v(q_L)]. \quad (5)$$

The benefit of type  $\theta_H$  deviating to  $r_L$ , captured by LHS of (5), consists of reduced payment,  $t_H - t_L$ , and its positive impact on the gain-loss utility,  $(1-p + p\lambda)(t_H - t_L)$ . To understand the latter, note first that the gain from paying  $t_L$  instead of  $t_H$  is weighted by probability  $1-p$  with which the buyer has expected the payment to be  $t_H$  according to the reference point. At the same time, by the deviation, the high type avoids the loss equal to  $\lambda(t_H - t_L)$  that he would have incurred from sticking with his equilibrium choice, which is weighted by probability  $p$  with which  $\theta_L$  would have occurred. The cost of deviation, captured by the RHS of (5), results from a reduced quality from  $q_H$  to  $q_L$  and can be explained similarly.

Notice also from (5) that higher  $\lambda$  amplifies both the benefit and cost of deviation. The relative magnitude is given by

$$B(p, \lambda) \equiv \frac{1 + (1-p) + p\lambda}{1 + p + (1-p)\lambda}. \quad (6)$$

Using this in the binding constraint (5), we obtain

$$t_H = t_L + \frac{\theta_H[v(q_H) - v(q_L)]}{B(p, \lambda)}. \quad (7)$$

Thus, if a higher  $\lambda$  makes  $B(p, \lambda)$  larger (smaller), then the loss aversion makes screening less (more) effective in enabling the extraction of more payment from the high type. We will refer to this as the *screening effect* of loss aversion; such an effect could be favorable or adverse to the seller.

Now, we describe the optimal screening menu and compare it with the optimal pooling menu.

**Proposition 3.** (a) *The optimal screening menu,  $\{(q_L^s, t_L^s), (q_L^s, t_H^s)\}$ , is such that*

$$\frac{c}{v'(q_L^s)} = \max \left\{ \frac{(\lambda + 1)B(p, \lambda)\theta_L - 2(1-p)\theta_H}{2pB(p, \lambda)}, 0 \right\} \quad (8)$$

$$\frac{c}{v'(q_H^s)} = \frac{\theta_H}{B(p, \lambda)}, \quad (9)$$

where  $q_L^s$ , if not equal to 0, increases in  $\lambda$  and  $q_H^s$  decreases (increases) in  $\lambda$  if  $p > \frac{1}{2}$  ( $p < \frac{1}{2}$ ).

(b) Any screening menu is dominated by the optimal pooling menu if and only if

$$\left(\frac{\lambda + 1}{2}\right) B(p, \lambda) \geq \frac{\theta_H}{\theta_L}, \quad (10)$$

which in turn holds if and only if  $\lambda \geq \lambda^S\left(p, \frac{\theta_H}{\theta_L}\right)$  for some threshold  $\lambda^S\left(p, \frac{\theta_H}{\theta_L}\right) > 1$  that decreases in  $p$  and increases in  $\frac{\theta_H}{\theta_L}$ .

In part (a) of Proposition 3, the optimal quality  $q_L$  increasing with  $\lambda$  should be expected from the participation effect. The behavior of  $q_H$  is related to the fact that  $B(p, \lambda)$  increases with  $\lambda$  if and only if  $p > \frac{1}{2}$ : That is, a higher  $\lambda$  means the adverse (favorable) screening effect if  $p > \frac{1}{2}$  ( $p < \frac{1}{2}$ ).

Part (b) states the condition under which pooling dominates screening. The inequality (10) holds when the participation effect, measured by  $\frac{\lambda+1}{2}$  (see (4) above), is large and/or when the screening effect works against the profitability of screening as  $B(p, \lambda)$  gets large. There are a couple of noteworthy observations here. First, the dominance of pooling over screening remains for high enough  $\lambda$  even when  $p < \frac{1}{2}$  such that the screening effect works favorably for the screening seller. This is because the participation effect dominates the screening effect, namely,  $\frac{\lambda+1}{2}$  increases with  $\lambda$  faster than  $B(p, \lambda)$  does. Second, the threshold,  $\lambda^S\left(p, \frac{\theta_H}{\theta_L}\right)$ , is decreasing in  $p$ , and this implies that screening is less attractive relative to pooling when the low type consumers are more abundant. This follows from the fact that a higher (ex ante) likelihood of  $\theta_L$  generates a greater deviation incentive for the high type via the gain-loss utility, i.e.  $B(p, \lambda)$  increases in  $p$ .

### 3.3 Reverse-Screening Menu

Let us consider next a reverse-screening menu  $R = \{r_L = (q_L, t_L), r_H = (q_H, t_H)\} \in \mathcal{M}^R$  such that  $q_L > q_H$  and  $t_L > t_H$ , satisfying the (IC) and (IR) constraints. The reverse-screening menu is a useful device to exploit the aforementioned participation effect by giving a higher

quality to the low type. Giving a higher quality to the low type, however, may create a deviation incentive for the high type. This incentive can be curbed should the high type suffer a sufficient loss from a higher deviation payment. How this loss is affected by the parameters in our model will determine when the reverse-screening menu is optimal.

We first provide a couple of necessary conditions for reverse-screening menu to be feasible or optimal.

**Lemma 1.** (a) *A reverse-screening menu can be a TPE only if*

$$\frac{\lambda + 1}{2} \geq \frac{\theta_H}{\theta_L}. \quad (11)$$

(b) *It is never optimal to offer a reverse-screening menu such that  $\theta_L v(q_L) > \theta_H v(q_H)$ .*

Part (a) states that loss aversion must be high enough to sustain a reverse-screening menu as a TPE. According to part (b), the seller does not want to reverse the qualities to the extent that the utility from quality consumption is reversed.

We now compare reverse-screening and pooling menus:<sup>13</sup>

**Proposition 4.** *Any reverse-screening menu is dominated by the optimal pooling menu if and only if*

$$\frac{1 + p + (1 - p)\lambda}{2} \leq \frac{\theta_H}{\theta_L}, \quad (12)$$

*which in turn holds if and only if  $\lambda \leq \lambda^R \left( p, \frac{\theta_H}{\theta_L} \right)$  for some threshold  $\lambda^R \left( p, \frac{\theta_H}{\theta_L} \right)$  that increases in  $p$  and  $\frac{\theta_H}{\theta_L}$ .*

The dominance of reverse-screening menu over pooling menu can be attributed to the participation effect that makes the increase in  $q_L$ , rather than  $q_H$ , more effective in extracting transfers. However, the profitability of reverse-screening menu can be limited by the incentive of the high type to deviate to the low type's bundle with high quality, depending on the degree of loss aversion and the proportion of low types. To see this, write and rearrange the  $(IC_H)$  constraint as

$$2\theta_H[v(q_L) - v(q_H)] \leq [1 + p + (1 - p)\lambda](t_L - t_H). \quad (13)$$

---

<sup>13</sup>For a detailed description of the optimal reverse-screening menu, refer to Lemma 5 in the Appendix.

The LHS and RHS of this inequality again represent the benefit and cost of  $\theta_H$  deviating to  $r_L$ , respectively. While the benefit side is not affected by the loss aversion,<sup>14</sup> the cost of deviation *is* affected, as indicated by the term  $[1 + p + (1 - p)\lambda]$  in (13), so that the high type is more inclined to deviate as  $\lambda$  gets lower and/or  $p$  gets higher.<sup>15</sup>

Combining Propositions 3 and 4, we observe that the reverse-screening menu can be optimal if  $\lambda$  is high enough. This peculiar phenomenon, however, seems confined to the case of binary (or very few) types since we later show that the reverse-screening menu can never be optimal with continuous types (Proposition 6). Also, our example in the next Section demonstrates that, even in the case of binary types, the optimality of reverse-screening requires a very large degree of loss aversion.

### 3.4 Optimal Menu

We are now ready to characterize the menu that maximizes the seller's expected profit among all TPE menus.

**Theorem 1.** *There exists some  $\bar{p} \in (0, 1)$  such that  $\lambda^S \left( p, \frac{\theta_H}{\theta_L} \right) \leq \lambda^R \left( p, \frac{\theta_H}{\theta_L} \right)$  if and only if  $p \geq \bar{p}$ . Then, the solution to [P] is given by*

- (a) a pooling menu if  $p \geq \bar{p}$  and  $\lambda \in [\lambda^S, \lambda^R]$ ;
- (b) a screening menu if  $\lambda < \min\{\lambda^R, \lambda^S\}$ ;
- (c) a reverse-screening menu if  $\lambda > \max\{\lambda^R, \lambda^S\}$ ;
- (d) either screening or reverse-screening menu if  $p < \bar{p}$  and  $\lambda \in [\lambda^R, \lambda^S]$ .

*Proof.* First, it is straightforward to see that

$$\lim_{p \rightarrow 0} \lambda^S = \infty > \frac{2\theta_H}{\theta_L} - 1 = \lim_{p \rightarrow 0} \lambda^R$$

---

<sup>14</sup>This is because the deviation to consume a larger quality  $q_L (> q_H)$  only adds to the gain in terms of the high type's utility from quality consumption. Note that the optimal reverse-screening menu must be such that  $\theta_H v(q_H) \geq \theta_L v(q_L)$ , that is, in equilibrium the high type derives a (weakly) greater utility than the low type from quality consumption.

<sup>15</sup>The intuition behind this comparative static with respect to  $p$  is that the high type does not find it too costly to deviate and pay the higher amount  $t_L$  when a large proportion of low type consumers are doing the same.

$$\lim_{p \rightarrow 1} \lambda^S = 2\sqrt{\frac{\theta_H}{\theta_L}} - 1 < \infty = \lim_{p \rightarrow 1} \lambda^R.$$

Thus, by the mean value theorem and the monotonicity of  $\lambda^S$  and  $\lambda^R$ , we can find  $\bar{p} \in (0, 1)$  such that  $\lambda^S \geq \lambda^R$  if and only if  $p \geq \bar{p}$ . Then, parts (a) to (d) of the claim immediately follow from combining part (b) of Proposition 3 and Proposition 4.  $\square$

Pooling can therefore be optimal if there is an enough mass of low types and the consumer is sufficiently, but not too, loss-averse. Otherwise, a screening or reverse-screening menu is optimal. In the latter case, there is a region of parameters, as shown in part (d), in which we have not been able to fully sort between screening and reverse-screening menus, but in most cases we expect the screening (reverse-screening) menu to be optimal if  $\lambda$  is low (high).

We illustrate the schedule of optimal menu in the following example. It shows that the pooling menu is optimal for a wide range of parameter values.<sup>16</sup>

**Example 1.** *Suppose that  $\theta_L/\theta_H = 1.5$ . The following Figure 1 divides the space of  $(\lambda, p)$  into four regions according to Theorem 1 and illustrates the type of optimal menu in each region.*

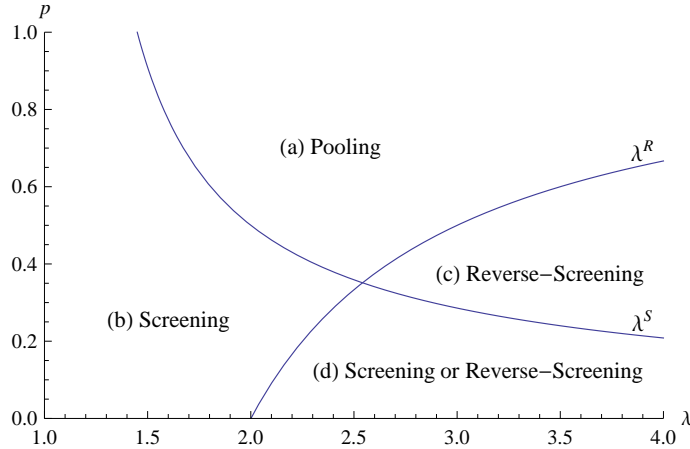


Figure 1

*It can be shown, though only numerically, that in the region (d), there is a threshold value of  $\lambda$  for each  $p$  below(above) which the screening(reverse-screening) menu is optimal.*

<sup>16</sup>Estimates of loss aversion have been obtained in a variety of contexts, ranging from 1.3 to 2.7. See Camerer (2006).

## 4 Discussion

### 4.1 The Role of Asymmetric Information

One can ask what role the informational asymmetry plays to generate the optimality of pooling and how it interacts with the loss aversion in doing so. As it will turn out, both loss aversion *and* informational asymmetry are indispensable for the optimality of pooling.

To highlight the role of informational asymmetry, we study a benchmark problem of a profit-maximizing monopolist who is symmetrically informed of  $\theta$  and thus need not be concerned about the incentive compatibility constraints. Specifically, let us consider the following problem [ $P^s$ ]:

$$\max_{\{(q_L, t_L), (q_H, t_H)\}} p(t_L - cq_L) + (1-p)(t_H - cq_H) \quad [P^s]$$

subject to  $R = \{(q_L, t_L), (q_H, t_H)\}$  satisfying  $(IR_L)$  and  $(IR_H)$ . Here, we assume that the symmetrically informed seller can commit to a menu ex ante such that she imposes  $(q_i, t_i)$  upon observing each type  $\theta_i$  being realized.

The following result gives a necessary condition for the optimal menu under symmetric information (see the Supplementary Material for proof).

**Lemma 2.** *The solution to [ $P^s$ ] must be such that  $\theta_H v(q_H) \geq \theta_L v(q_L)$  and  $t_H \geq t_L$ .*

Using the above Lemma and the fact that both  $(IR)$  constraints are binding yields

$$t_L = \frac{(\lambda + 1)}{2} \theta_L v(q_L) \quad \text{and} \quad t_H = t_L + \frac{\theta_H v(q_H) - \theta_L v(q_L)}{B(p, \lambda)}. \quad (14)$$

Assuming  $\theta_H v(q_H) > \theta_L v(q_L)$  at the optimum,<sup>17</sup> we can plug (14) into the objective function and take the first-order conditions to obtain

$$\frac{c}{v'(q_L^*)} = \frac{[(\lambda + 1)B(p, \lambda) - 2(1 - p)] \theta_L}{2pB(p, \lambda)} \quad (15)$$

$$\frac{c}{v'(q_H^*)} = \frac{\theta_H}{B(p, \lambda)}. \quad (16)$$

Note from (15) and (16) that  $q_L^* \geq q_H^*$  if and only if

$$\frac{(\lambda + 1)B(p, \lambda) - 2(1 - p)}{2p} \geq \frac{\theta_H}{\theta_L}, \quad (17)$$

---

<sup>17</sup>It is possible to have  $\theta_H v(q_H) = \theta_L v(q_L)$  at the optimum, which case we ignore to ease the exposition.

which holds for  $\lambda$  exceeding some threshold since  $(\lambda+1)B(p, \lambda)$  strictly increases in  $\lambda$  without bound. Thus, when symmetrically informed with  $\lambda$  high enough to satisfy (17), the seller can maximize profit by endowing the low type with a higher quality but charging the high type with a larger transfer (see (14)). Note that the optimal qualities are the same when (17) holds as equality, which obviously is a knife-edge phenomenon. Furthermore, the same quality does not necessarily mean the same transfer.

This implies that pooling does not arise when the buyers are loss averse but do not hold private information. Neither does it emerge as a consequence of asymmetric information alone, as Mussa and Rosen (1978) show. The optimality of pooling is indeed a consequence of the interplay between loss aversion and asymmetric information. From the above analysis, we can see that pooling emerges as the optimal menu when the quality reversal is desirable due to loss aversion but is not feasible in the presence of asymmetric information.

## 4.2 Ex-Ante Participation and Robustness of Pooling

The optimal menu described in Section 3 has the feature that the buyer often incurs an ex-ante loss, that is, the buyer's ex-ante expected utility (including anticipated gain-loss) falls below zero. This can be problematic for the seller since the optimal menu may admit alternative PEs that yield higher payoffs to the buyer, especially, one that induces no participation; even without multiple PEs, the buyer may find some commitment device to stay away from the menu altogether. We next show that the dominance of pooling over separating menu continues to hold when the optimal menu is modified to accommodate the buyer's ex-ante participation incentive.

Given a menu  $R = \{(q_L, t_L), (q_H, t_H)\}$ , let  $U(R)$  denote the buyer's ex-ante expected utility, and this is given by

$$U(R) \equiv pu(r_L; \theta_L, R) + (1 - p)u(r_H; \theta_H, R). \quad (18)$$

We now consider the following program

$$\max_{\{(q_L, t_L), (q_H, t_H)\}} p(t_L - cq_L) + (1 - p)(t_H - cq_H) \quad [P']$$

subject to  $R = \{(q_L, t_L), (q_H, t_H)\}$  being a TPE and satisfying an extra constraint, which we shall call the *ex-ante participation constraint*, or simply *(EA)*,

$$U(R) \geq 0. \tag{EA}$$

Comparing screening and pooling menus under this program yields our next result.<sup>18</sup>

**Proposition 5.** *The set of parameters under which pooling dominates screening is larger with  $[P']$  than with  $[P]$ .*

A negative ex-ante utility in the solution to  $[P]$  arises in part because the optimal menu is designed to exploit the buyer's loss aversion to extract more payment, especially from the low type as indicated by the participation effect mentioned earlier. Assuming for instance that  $p$  is close 1, the buyer's ex-ante utility consists mostly of the low type's intrinsic utility, which, given the transfer  $t_L$  in (4), is equal to  $(1 - \frac{\lambda+1}{2})\theta_L v(q_L) < 0$ .<sup>19</sup> From this, one might reason that the ex-ante loss problem could be handled by reducing the low type's transfer/quality, which would in turn restore the optimality of screening menu.

What is also responsible for the ex-ante loss problem, however, is the gap between the two types' quality/transfer schedules in the screening menu that worsens the gain-loss utility and thereby reduces the ex-ante utility. The pooling menu alleviates this effect better than the screening menu. Indeed, Proposition 5 shows that the seller chooses pooling over screening more often when trying to avoid the ex-ante loss problem.

The next example shows that the expansion of parameter range where pooling dominates screening can be quite stark.

**Example 2.** *This example is a repetition of Example 1. To the right of locus  $EE$ , screening menus are dominated by a pooling menu when *(EA)* is imposed.<sup>20</sup>*

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<sup>18</sup>See the Supplementary Material for its proof.

<sup>19</sup>One can show that the optimal pooling or reverse-screening characterized in Section 3 menu always involves ex-ante loss for the buyer.

<sup>20</sup>How we obtain  $EE$  is described in the proof of Proposition 5 in the Supplementary Material.

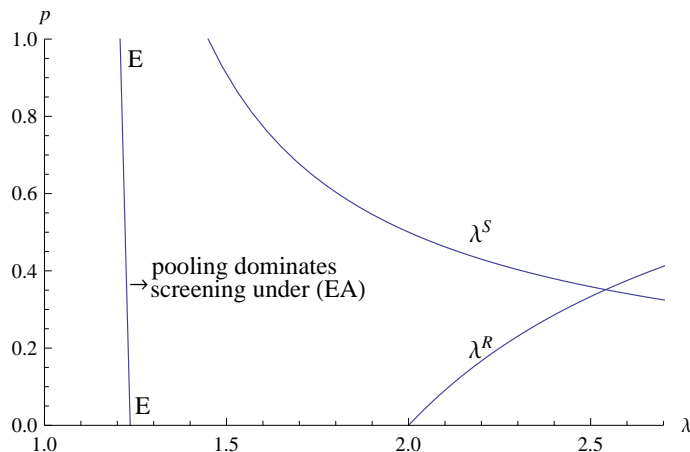


Figure 2

## 5 Optimal Menu with Continuous Types

We now extend our analysis to the case in which there is a continuum of consumer types over the interval  $[\underline{\theta}, \bar{\theta}]$ . The type distribution is given by a cdf  $F$ , which has a strictly positive and continuously differentiable pdf  $f$ . Define the “virtual value” function as

$$J(\theta) \equiv \theta - \frac{1 - F(\theta)}{f(\theta)},$$

and assume that it is strictly increasing. As is well known, without loss aversion, this assumption leads to the full separation of types (i.e. the optimal quality schedule that is strictly increasing where the virtual value is not negative). Therefore, this assumption will enable us to capture whether loss aversion causes any pooling to arise in the optimal menu.

Let  $(q, t) : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}_+ \times \mathbb{R}$  denote a menu that constitutes a TPE. In order to facilitate a characterization of the optimal menu with continuous types, we shall restrict our attention to the set of menus in which  $q(\cdot)$  is nondecreasing and thus  $t(\cdot)$  nondecreasing as well. Our analysis of the binary type case above indeed suggests that non-monotonic menus could be feasible as TPE menus or even be optimal. However, the monotonicity restriction makes the analysis tractable as, without any such restriction, the gain-loss utility can depend on the detailed pattern of quality/transfer schedule in a complex way beyond the scope of this analysis. At the end of this section, we shall show that a particular type of reverse-screening, in which the quality schedule is decreasing while the utility from quality is non-decreasing,

cannot be optimal. For further simplicity, we assume that  $q(\cdot)$  and  $t(\cdot)$  are continuous.<sup>21</sup>

Given the monotonicity of  $q(\cdot)$ ,  $V(\theta) \equiv \theta v(q(\theta))$  is strictly increasing and we can thus define

$$\hat{\theta}(\theta, \theta') \equiv \sup\{r \in [\underline{\theta}, \bar{\theta}] \mid V(s) \leq \theta v(q(\theta')), \forall s \leq r\}.$$

Then, the payoff of type  $\theta$  from announcing  $\theta'$  is given by

$$\begin{aligned} U(\theta'; \theta) = \theta v(q(\theta')) - t(\theta') + & \left[ \int_{\underline{\theta}}^{\hat{\theta}(\theta, \theta')} (\theta v(q(\theta')) - V(s)) dF(s) + \int_{\theta'}^{\bar{\theta}} (t(s) - t(\theta')) dF(s) \right] \\ & - \lambda \left[ \int_{\hat{\theta}(\theta, \theta')}^{\bar{\theta}} (V(s) - \theta v(q(\theta'))) dF(s) + \int_{\underline{\theta}}^{\theta'} (t(\theta') - t(s)) dF(s) \right], \end{aligned} \quad (19)$$

where the expression in the first (second) square bracket corresponds to the gain (loss). The (IC) constraint for TPE can be written as

$$U(\theta) \equiv U(\theta; \theta) = \max_{\theta' \in [\underline{\theta}, \bar{\theta}]} U(\theta'; \theta), \forall \theta, \quad (20)$$

while the (IR) constraint as

$$U(\theta) \geq \int_{\underline{\theta}}^{\bar{\theta}} (t(s) - \lambda V(s)) dF(s), \forall \theta. \quad (21)$$

In the following step, we express the payment schedule  $t(\cdot)$  in terms of the quality schedule  $q(\cdot)$ , as often done in the standard screening model. Note first that, using (19), we obtain

$$\begin{aligned} \frac{\partial U(\theta'; \theta)}{\partial \theta} &= v(q(\theta')) + \int_{\underline{\theta}}^{\hat{\theta}(\theta, \theta')} v(q(\theta')) dF(s) + \lambda \int_{\hat{\theta}(\theta, \theta')}^{\bar{\theta}} v(q(\theta')) dF(s) \\ &= v(q(\theta')) \left[ 1 + F(\hat{\theta}(\theta, \theta')) + \lambda(1 - F(\hat{\theta}(\theta, \theta'))) \right]. \end{aligned} \quad (22)$$

Given this, and since  $\hat{\theta}(\theta, \theta) = \theta$ , the envelope theorem implies

$$U'(\theta) = \left. \frac{\partial U(\theta'; \theta)}{\partial \theta} \right|_{\theta'=\theta} = v(q(\theta))(1 + F(\theta) + \lambda(1 - F(\theta))). \quad (23)$$

An alternative way to obtain  $U'(\cdot)$  is to set  $\theta' = \theta$  in (19) and differentiate the resulting expression with  $\theta$ , which yields

$$U'(\theta) = V'(\theta) - t'(\theta) + V'(\theta)F(\theta) - t'(\theta)(1 - F(\theta)) - \lambda[-V'(\theta)(1 - F(\theta)) + t'(\theta)F(\theta)]$$

---

<sup>21</sup>If the optimal schedule involves some jump(s), then it will manifest itself as a boundary solution of the optimization program since any such schedule can be approximated by a sequence of continuous schedules.

$$\begin{aligned}
&= V'(\theta) (1 + F(\theta) + \lambda(1 - F(\theta))) - t'(\theta) (2 - F(\theta) + \lambda F(\theta)) \\
&= (v(q(\theta)) + \theta(v(q(\theta))))' (1 + F(\theta) + \lambda(1 - F(\theta))) - t'(\theta) (2 - F(\theta) + \lambda F(\theta)).
\end{aligned}$$

Equating this with (23) yields

$$t'(\theta) = \frac{\theta(v(q(\theta)))'(1 + F(\theta) + \lambda(1 - F(\theta)))}{1 + (1 - F(\theta)) + \lambda F(\theta)} = (v(q(\theta)))'G(\theta, \lambda), \quad (24)$$

where

$$G(\theta, \lambda) \equiv \theta \frac{(1 + F(\theta) + \lambda(1 - F(\theta)))}{1 + (1 - F(\theta)) + \lambda F(\theta)} = \theta H(\theta, \lambda).$$

Note that the function  $H(\theta, \lambda)$  is the continuous type counterpart of the inverse of  $B(p, \lambda)$  in (7) in that it affects the speed with which the payment increases as the consumer's type, and thus its corresponding quality, marginally increases. Without reference-dependent utility, the rate of increase is proportional to  $G(\theta, 1) = \theta$ ; this should be adjusted using  $H(p, \lambda)$  in the presence of reference-dependent utility.

The numerator of  $G(\theta, \lambda)$  measures the benefit of this marginal change due to the gain-loss utility: the gain, which  $\theta$  enjoys relative to the types below, increases by  $\theta(v(q(\theta)))'F(\theta)$  while the loss, which  $\theta$  suffers relative to the types above, decreases by  $\lambda\theta(v(q(\theta)))'(1 - F(\theta))$ . So, the marginal benefit is proportional to  $[1 + F(\theta) + \lambda(1 - F(\theta))]$ . This benefit does not however translate entirely into the payment increase since a higher payment negatively impacts the gain-loss utility. If the payment increases by  $t'(\theta)$  for type  $\theta$ , then the gain, which  $\theta$  enjoys relative to the types above, decreases by  $t'(\theta)(1 - F(\theta))$  while the loss, which  $\theta$  suffers relative to the types below, increases by  $\lambda t'(\theta)F(\theta)$ . To account for this impact, the payment increase associated with the quality increase should be adjusted downward as much as the denominator of  $G(\theta, \lambda)$ .

Given this interpretation, we refer to  $G(\theta, \lambda)$  as the “gain-loss adjusted type,” whose behavior will turn out to be crucial for determining the optimal quality schedule. Note that  $G(\theta, \lambda) > \theta$  if  $\theta < F^{-1}(\frac{1}{2})$  (and  $G(\theta, \lambda) < \theta$  if  $\theta > F^{-1}(\frac{1}{2})$ ), so the gain-loss adjusted type is leveled out. Moreover,  $H(\theta, \lambda)$  decreases in  $\theta$  and does so faster with higher  $\lambda$ , which may cause  $G(\theta, \lambda) = \theta H(\theta, \lambda)$  to decrease.

The equation in (24) results from the local incentive compatibility condition. While the global incentive compatibility is usually guaranteed by the nonnegative cross derivative

of  $U(\theta'; \theta)$ , the expression in (22) may not be increasing with  $\theta$  since  $\hat{\theta}(\theta, \theta')$  is increasing with  $\theta$  and thus the term in the square bracket,  $1 + F(\hat{\theta}(\theta, \theta')) + \lambda(1 - F(\hat{\theta}(\theta, \theta')))$ , is decreasing. We introduce the following assumption that is sufficient to ensure the global incentive compatibility:

**Assumption 1.**  $\theta(1 + F(\theta) + \lambda(1 - F(\theta)))$  is non-decreasing in  $\theta$ .

**Lemma 3.** Suppose that Assumption 1 holds. Given any non-decreasing quality schedule  $q(\cdot)$  and the payment schedule satisfying (24), the (global) (IC) constraint (20) is satisfied.<sup>22</sup>

Note that if the menu is fully pooling i.e.  $q(\cdot)$  and  $t(\cdot)$  are constant, then the (IC) constraint is trivially satisfied so Assumption 1 is unnecessary.

We now turn to the analysis of finding the optimal menu. Given (24), the seller's revenue can be expressed as follows: letting  $G_\theta(\theta, \lambda) \equiv \frac{\partial G(\theta, \lambda)}{\partial \theta}$ ,

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} t(\theta) dF(\theta) &= \int_{\underline{\theta}}^{\bar{\theta}} \left( \int_{\underline{\theta}}^{\theta} t'(s) ds + t(\underline{\theta}) \right) dF(\theta) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \left( \int_{\underline{\theta}}^{\theta} (v(q(s)))' G(s, \lambda) ds \right) dF(\theta) + t(\underline{\theta}) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \left( v(q(\theta)) G(\theta, \lambda) - \int_{\underline{\theta}}^{\theta} v(q(s)) G_\theta(s, \lambda) ds \right) dF(\theta) + t(\underline{\theta}) - v(q(\underline{\theta})) G(\underline{\theta}, \lambda) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} v(q(\theta)) \left( G(\theta, \lambda) - \frac{1 - F(\theta)}{f(\theta)} G_\theta(\theta, \lambda) \right) dF(\theta) + t(\underline{\theta}) - \underline{\theta} v(q(\underline{\theta})) \frac{(1 + \lambda)}{2} \\
&= \int_{\underline{\theta}}^{\bar{\theta}} v(q(\theta)) J(\theta, \lambda) dF(\theta) + t(\underline{\theta}) - \underline{\theta} v(q(\underline{\theta})) \frac{(1 + \lambda)}{2}, \tag{25}
\end{aligned}$$

where

$$J(\theta, \lambda) \equiv G(\theta, \lambda) - \frac{1 - F(\theta)}{f(\theta)} G_\theta(\theta, \lambda),$$

and the second equality follows from integration by parts as does the third equality along with the fact that  $G(\underline{\theta}, \lambda) = (\frac{1+\lambda}{2})\underline{\theta}$ .

Let us refer to  $J(\theta, \lambda)$  as “gain-loss adjusted virtual value,” which boils down to the usual virtual value  $J(\theta)$  if  $\lambda = 1$ . The next result says that the transfer must be designed in such a way that the participation constraint is binding at the lowest type  $\underline{\theta}$ .

<sup>22</sup>See the Supplementary Material for the proof.

**Lemma 4.** *At the optimal menu, it must be that*

$$t(\theta) = \left(\frac{1+\lambda}{2}\right)\theta v(q(\theta)), \quad (26)$$

which implies that the (IR) constraint (21) is satisfied and, moreover, binding at  $\theta = \underline{\theta}$ .

Thus, the seller's problem, denoted by  $[P^c]$ , can be written as

$$\max_{q(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} [v(q(\theta))J(\theta, \lambda) - cq(\theta)] dF(\theta) \quad [P^c]$$

subject to  $q(\cdot)$  being nondecreasing.<sup>23</sup> The shape of the optimal quality schedule will depend on the behavior of  $J(\cdot, \lambda)$ , the gain-loss adjusted virtual value. While  $J(\cdot, \lambda)$  may behave in a complicated way depending on the distribution  $F(\cdot)$ , it can be decreasing over some or entire range for high enough  $\lambda$ , which implies that the optimal menu must involve some pooling.

To see this, we can obtain

$$J_{\theta}(\theta, \lambda) \equiv \frac{\partial J(\theta, \lambda)}{\partial \theta} = J'(\theta)G_{\theta}(\theta, \lambda) - \frac{1-F(\theta)}{f(\theta)}G_{\theta\theta}(\theta, \lambda), \quad (27)$$

where  $G_{\theta\theta}(\theta, \lambda) \equiv \frac{\partial G(\theta, \lambda)^2}{\partial^2 \theta}$ . Two important terms here are  $G_{\theta}(\theta, \lambda)$  and  $G_{\theta\theta}(\theta, \lambda)$ . As mentioned above, it is possible to have  $G_{\theta}(\theta, \lambda) < 0$  as  $\lambda$  increases. Given this, (27) tells that  $J(\theta, \lambda)$  can decrease if  $G_{\theta\theta}(\theta, \lambda) > 0$ . Note that the last term of (27) concerns the impact of loss aversion on the *information rent*.<sup>24</sup> While  $G_{\theta\theta}(\theta, 1) = 0$ , having  $G_{\theta\theta}(\theta, \lambda) > 0$  with  $\lambda > 1$  means that loss aversion aggravates the information rent problem, which may cause the gain-loss adjusted virtual value to decrease and, hence, some pooling to arise. Indeed, this can happen if  $\lambda$  is high enough, as the following result shows:

**Theorem 2.** *Consider menus with non-decreasing  $q(\cdot)$ . Then, the optimal menu has the following properties:*

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<sup>23</sup>Note that global incentive compatibility is ignored here. It can be verified by showing that Assumption 1 holds, or checking directly the optimality of  $\theta' = \theta$  in the (IC) constraint (20) by plugging in the solution of  $[P^c]$ . Also, if the optimal schedule turns out to be constant, the global incentive compatibility is trivially satisfied.

<sup>24</sup>In the standard screening model, the expression  $\frac{1-F(\theta)}{f(\theta)}$  represents the information rents that have to be given up to the types above  $\theta$  if an extra unit of good is to be sold to  $\theta$ .

- (a) Suppose Assumption 1 holds and  $\frac{\lambda^2+2\lambda-3}{2(\lambda+1)} > \frac{1}{\theta f(\theta)}$ . Then, pooling occurs around the highest type  $\bar{\theta}$ ;
- (b) Suppose  $\underline{\theta} > 0$ ,  $\theta f(\theta) > F(\theta) \forall \theta$ , and  $f'(\theta) \leq 0 \forall \theta$ . Then, there exists some  $\bar{\lambda} > 1$  such that, for any  $\lambda > \bar{\lambda}$ , pooling occurs over the entire interval  $[\underline{\theta}, \bar{\theta}]$ .

The inequality condition in part (a) above is equivalent to requiring that  $G_{\theta}(\bar{\theta}, \lambda) < 0$ , i.e. the gain-loss adjusted type decreases with the original type around the top. Without having to concern with information rent at the top, this means that the gain-loss adjusted virtual value also decreases, leading to pooling at the top. Note that the inequality in (a) never holds if  $\lambda = 1$ .

Part (b) gives a set of conditions sufficient for full pooling to be optimal. The first condition,  $\underline{\theta} > 0$ , prevents the optimal menu from excluding the bottom type, as required by a full pooling menu. To understand the second condition, let us first note

$$\lim_{\lambda \rightarrow \infty} G(\theta, \lambda) = \theta \frac{1 - F(\theta)}{F(\theta)}.$$

Thus, for sufficiently high  $\lambda$ , the gain-loss adjusted type decreases going from  $\underline{\theta}$  to  $\bar{\theta}$  while it may not in-between. Then, the condition that  $\theta f(\theta) > F(\theta) \forall \theta$  ensures that this expression monotonically decreases over the entire interval so that  $G_{\theta}(\theta, \lambda)$  is always negative for sufficiently high  $\lambda$ . The last condition,  $f'(\theta) \leq 0$ , ensures (along with the second condition) that  $G_{\theta\theta}(\theta) \leq 0$  for sufficiently high  $\lambda$ , which means worsening of the information rent problem due to loss aversion. Note that this condition is consistent with the observation in the previous binary type analysis that the screening effect adversely affects the profitability of a screening menu when the low type is abundant.

The following examples demonstrate that  $\lambda$  needs not be very high in order for some or full pooling to arise and also that a diverse pattern of pooling can emerge depending on the value distribution.

**Example 3.** Suppose that  $\theta$  is uniformly distributed on  $[1, 2]$  so  $F(\theta) = \theta - 1$ .<sup>25</sup> The following Figure 3 draws  $J(\cdot, \lambda)$  in the left panel and its “ironing-out” obtained using the technique of Myerson (1981) or Toikka (2009) in the right panel:

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<sup>25</sup>One can verify that Assumption 1 holds if  $\lambda \leq 2$ .

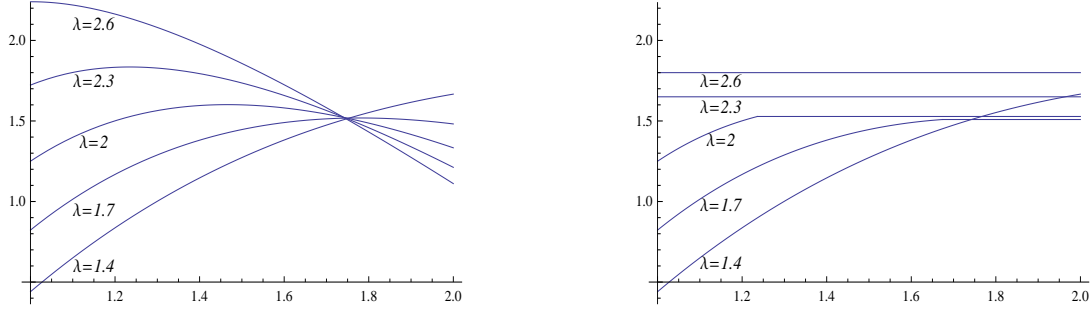


Figure 3

Pooling does not arise if  $\lambda = 1.4$ , and does arise in the interval  $[1.674, 2]$  if  $\lambda = 1.7$ , in the larger interval  $[1.236, 2]$  if  $\lambda = 2$  and over the entire interval if  $\lambda = 2.3$  or higher.

**Example 4.** Suppose that  $\theta$  is distributed on the interval  $[1, 2]$  with  $F(\theta) = 1 - (2 - \theta)^n$  for  $n \geq 1$ . Note that, as  $n$  grows,  $F(\cdot)$  becomes more concave so that weights are shifting toward lower types. The following Figure 4 illustrates  $J(\cdot, 1.75)$  in the left panel and its ironing in the right panel:

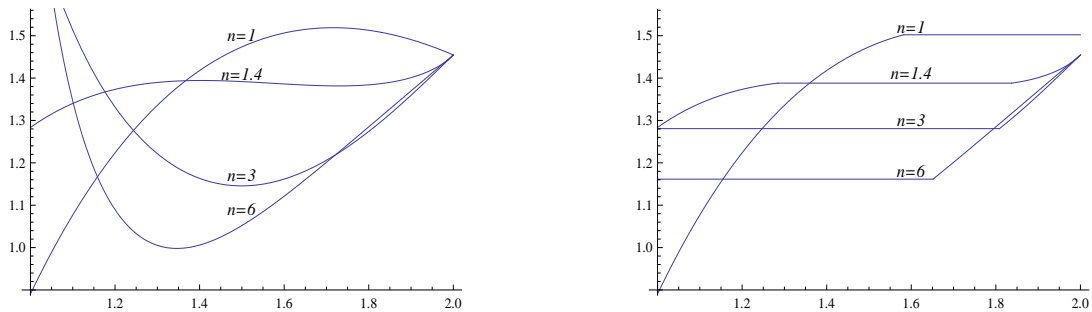


Figure 4

So pooling occurs in the upper interval for  $n = 1$ , in the middle interval for  $n = 1.4$ , and in the lower interval if  $n = 3$  or  $6$ . As  $n$  grows, a pooling area shifts down as probability weights do. This can be understood from the fact that screening types with higher probability weights has a greater impact on the gain-loss utility.

Our characterization of the optimal menu in this section has been obtained under the assumption of the non-decreasing quality. While our results highlight the difficulty that the requirements of TPE generate on conventional screening, pooling may still be dominated by other types of menus. We finish our analysis by comparing a pooling menu with a particular type of reverse-screening menu in which  $q(\theta)$  is decreasing but  $V(\theta) = \theta q(\theta)$  is increasing.

**Proposition 6.** *Any reverse-screening menu with a decreasing  $q(\cdot)$  and increasing  $V(\cdot)$  is dominated by a pooling menu with constant  $q(\cdot)$ .*

The key to understanding this result is that given the reverse-screening menu, the gain-loss utility works in the same direction for two accounts, quality and transfer. That is, the utility gain for each type  $\theta$  in terms of both quality and transfer comes from the comparison with the types below. This makes the formula for the marginal transfer in (24) change to the standard one,

$$t'(\theta) = (v(q(\theta)))'\theta, \tag{28}$$

which means that the virtual value function remains the same as the standard form. So the reversing of quality schedule is never optimal given that the virtual value is increasing. Interestingly, this result contrasts with Theorem 1 in the binary case that the reverse-screening menu is optimal for some parameter values. In the binary case, the revenue impact of quality increase for type  $\theta_L$  is more than marginal and moreover depends on the degree of loss aversion. By contrast, in the continuous case, the loss aversion does not play any role in determining the (marginal) relationship between quality and transfer schedules, as shown in (28). Note that the result in Proposition 6 does not rule out a possibility of reverse-screening over some subinterval of types at the optimal menu, whose analysis is beyond the scope of this paper and left for future research.

## 6 Conclusion

We often find sellers offering menus with just a small number of bundles. This paper demonstrates that such observations are consistent with profit-maximizing firms that face loss averse consumers. We show that, in the binary type case, a pooling menu is the seller's optimal menu under a range of loss aversion parameter if the low willingness-to-pay consumers are sufficiently abundant. This result arises as a consequence of the interplay between loss aversion and asymmetric information. We also identify conditions under which partial or even full pooling dominates screening for the seller facing a continuum of consumer types.

The analysis of price discrimination with loss averse consumers opens up some new research questions. In particular, our results suggest several potentially interesting channels of empirical study. For instance, since the optimality of pooling requires a sufficiently large mass of consumers with low demand, one may explore if indeed coarse screening occurs more often when the demand estimates point to more consumers with low willingness to pay. Another suggestion from our model is that reverse-screening can be optimal if the consumer is significantly loss averse, but this is difficult to justify in practice. Measuring the degree of consumer loss aversion in price discrimination settings may shed insights into this dichotomy.

Our model abstracts from many aspects that might be present in reality. First, the degree of loss aversion may well vary among different consumers. The seller may then be able to employ the varying magnitudes of loss aversion as an extra screening device. An analysis of optimal menu in this case could be interesting but challenging since it will involve taking on the multidimensional screening problem. Second, there are factors other than willingness to pay that influence the form of optimal menu (for instance, liquidity constraints as considered by Che and Gale (2000)). How those factors interact with loss aversion in determination of optimal menus poses another interesting avenue for future research. Finally, there could be a role for the use of random contracts. Heidhues and Köszegi (2005) discuss how a monopolist seller facing loss averse consumers may be able to employ random pricing to eliminate multiple personal equilibria and increase profits, but their argument is restricted to a deterministic environment. Our setup is different and requires an analysis of random contracts in the presence of uncertainty.

## Appendix

***Proof of Proposition 3:*** We maximize the seller's profit given in [P] under the (IC) and (IR) constraints and under the inequality constraint,  $q_H - q_L \geq 0$ . We show the following: When the constraint  $q_H - q_L \geq 0$  is not binding, the optimal qualities must be given by (8) and (9); If (10) holds, then the constraint is binding so any screening menu is dominated by the optimal pooling menu.

First, one can easily check that  $(IR_H)$  is implied by  $(IR_L)$  and  $(IC_H)$  since

$$u(r_H|\theta_H, R) \geq u(r_L|\theta_H, R) \geq u(r_L|\theta_L, R) \geq u(\emptyset|\theta_L, R) = u(\emptyset|\theta_H, R), \quad (\text{A.1})$$

where the second inequality holds since if two types choose the same bundle,  $r_L$ , then  $\theta_H$  is better off in terms of both intrinsic and gain-loss utilities. Next, we can write  $(IC_L)$  and  $(IC_H)$  as

$$[1 + p + (1 - p)\lambda]\theta_L[v(q_H) - v(q_L)] \leq [1 + (1 - p) + p\lambda](t_H - t_L) \leq [1 + p + (1 - p)\lambda]\theta_H[v(q_H) - v(q_L)].$$

By the usual argument,  $(IR_L)$  and  $(IC_H)$  must be binding.<sup>26</sup> Given the binding  $(IC_H)$  and the above equations for the  $(IC)$  constraints,  $(IC_L)$  is satisfied given the constraint  $q_H \geq q_L$ .

Using the two binding constraints, we obtain (4) and (7) for  $t_L$  and  $t_H$ , respectively. We substitute these into the objective function and maximize it, ignoring the inequality constraint,  $q_H - q_L \geq 0$ . Then, the first order conditions with respect to  $q_L$  and  $q_H$  yield (8) and (9). Given this, one can check  $v'(q_H) \geq v'(q_L)$  if and only if the inequality (10) holds, which means that if (10) holds then the ignored constraint is binding at the optimum, that is, any screening menu is dominated by the optimal pooling menu.

To obtain the comparative statics for  $q_L$ ,  $q_H$ , and  $\lambda^S$ , let us first observe the following facts: (i)  $B(p, \lambda)$  increases with  $\lambda > 1$  if and only if  $p > \frac{1}{2}$ ; (ii)  $B(p, \lambda)$  increases with  $p$  if  $\lambda > 1$ ; and (iii)  $(\lambda + 1)B(p, \lambda)$  increases from 1 to infinity as  $\lambda$  increases starting from  $\lambda = 1$ . The comparative statics for  $q_H$  directly follows from (i) and the fact that  $\frac{c}{v'(c)}$  is increasing. As for the comparative statics regarding  $q_L$ , rewrite the maximand in (8) as

$$\frac{\theta_L - [2(1 - p)\theta_H] / [(\lambda + 1)B(p, \lambda)]}{2p/(\lambda + 1)},$$

whose numerator increases with  $\lambda$  by (iii), while its denominator decreases. So the optimal  $q_L$ , if not equal to 0, must increase with  $\lambda$ . The existence and properties of  $\lambda^S(p, \theta_H/\theta_L)$  follow from (ii) and (iii).  $\square$

Before proving Lemma 1 and Proposition 4, we write here the  $(IC)$  and  $(IR)$  constraints for the reverse-screening menu, whose forms differ depending whether  $\theta_L v(q_L) \geq \theta_H v(q_H)$  or  $\theta_L v(q_L) \leq \theta_H v(q_H)$ . (For the full expressions, see the Supplementary Material.)

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<sup>26</sup>If  $(IR_L)$  is not binding, then the seller can slightly increase both  $t_L$  and  $t_H$  by the same amount. If  $(IC_H)$  is not binding, then the seller can increase  $t_H$  slightly.

In case  $\theta_L v(q_L) \geq \theta_H v(q_H)$ ,

$$[1 + p + (1 - p)\lambda](t_L - t_H) \geq [1 + (1 - p) + p\lambda]\theta_H(v(q_L) - v(q_H)) - p(\lambda - 1)(\theta_H - \theta_L)v(q_L) \quad (IC'_H)$$

$$[1 + (1 - p) + p\lambda]\theta_L(v(q_L) - v(q_H)) + (1 - p)(\lambda - 1)(\theta_H - \theta_L)v(q_H) \geq [1 + p + (1 - p)\lambda](t_L - t_H) \quad (IC'_L)$$

$$\theta_H(\lambda + 1)v(q_H) \geq 2t_H \quad (IR'_H)$$

$$[1 + (1 - p) + p\lambda]\theta_L v(q_L) + (1 - p)(\lambda - 1)\theta_H v(q_H) \geq [1 + p + (1 - p)\lambda]t_L - (1 - p)(\lambda - 1)t_H \quad (IR'_L)$$

while in case  $\theta_L v(q_L) \leq \theta_H v(q_H)$ ,

$$[1 + p + (1 - p)\lambda](t_L - t_H) \geq 2\theta_H[v(q_L) - v(q_H)] \quad (IC''_H)$$

$$(\lambda + 1)\theta_L[v(q_L) - v(q_H)] \geq [1 + p + (1 - p)\lambda](t_L - t_H) \quad (IC''_L)$$

$$[1 + p + (1 - p)\lambda]\theta_H v(q_H) + p(\lambda - 1)\theta_L v(q_L) \geq 2t_H \quad (IR''_H)$$

$$(\lambda + 1)\theta_L v(q_L) \geq [1 + p + (1 - p)\lambda]t_L - (1 - p)(\lambda - 1)t_H. \quad (IR''_L)$$

**Proof of Lemma 1:** To show (a), let us consider both cases of reverse-screening menu. In case  $\theta_L v(q_L) \leq \theta_H v(q_H)$ , the LHS of  $(IC''_L)$  being greater than the RHS of  $(IC''_H)$  yields (11) after rearrangement. In case  $\theta_L v(q_L) \geq \theta_H v(q_H)$ , combining  $(IC'_L)$  and  $(IC'_H)$  yields

$$(\lambda + 1)v(q_H) - 2v(q_L) \geq 0, \quad (A.2)$$

which implies

$$\frac{\lambda + 1}{2} \geq \frac{v(q_L)}{v(q_H)} \geq \frac{\theta_H}{\theta_L}.$$

To prove (b), consider the problem of maximizing the seller's profit under the  $(IC')$  and  $(IR')$  constraints and the constraint that  $\theta_L v(q_L) \geq \theta_H v(q_H)$ . We want to show that this last constraint binds at the optimum.

First, the same inequalities as in (A.1) can be used to show that  $(IR'_H)$  is implied by  $(IC'_H)$  and  $(IR'_L)$ . Next,  $(IR'_L)$  must be binding at the optimum since otherwise  $t_H$  and  $t_L$  could be slightly increased by the same amount in a way that all the constraints continue to hold. Also, either  $(IC'_H)$  or  $(IC'_L)$  must be binding since otherwise one could increase  $t_H$

by small  $\epsilon > 0$  and  $t_L$  by  $\frac{(1-p)(\lambda-1)}{1+p+(1-p)\lambda}\epsilon$ , and still satisfy all the constraints. Using two binding constraints,  $(IR'_L)$  and one of the two  $(IC')$  constraints, we can express  $t_L$  and  $t_H$  in terms of  $q_L$  and  $q_H$ , which we denote as  $t_L(q_L, q_H)$  and  $t_H(q_L, q_H)$ .

Given that one of the two  $(IC')$  constraints is binding, the other  $(IC')$  constraint is satisfied if and only if (A.2) is satisfied so we can replace the latter  $(IC')$  constraint by the inequality constraint (A.2). Now, using (A.2) and  $\theta_L v(q_L) - \theta_H v(q_H) \geq 0$  as constraints, the Lagrangian for the seller's maximization problem can be written as

$$\begin{aligned} \mathcal{L}(q_L, q_H, \gamma, \mu) = & p(t_L(q_L, q_H) - cq_L) + (1-p)(t_H(q_L, q_H) - cq_H) \\ & + \gamma[(\lambda+1)v(q_H) - 2v(q_L)] + \mu[\theta_L v(q_L) - \theta_H v(q_H)], \end{aligned} \quad (\text{A.3})$$

where  $\gamma$  and  $\mu$  are nonnegative multipliers. We show that  $\theta_L v(q_L) - \theta_H v(q_H) = 0$  at the optimum. Supposing not, or  $\mu = 0$ , we establish a contradiction in case  $(IC'_L)$  is binding. In case  $(IC'_H)$  is binding, an analogous proof applies, which is thus omitted.

If  $(IC'_L)$  and  $(IR'_L)$  are binding, we then obtain

$$\begin{aligned} t_H(q_L, q_H) &= \frac{(\lambda+1)\theta_H v(q_H)}{2} - 2(\theta_H - \theta_L)v(q_L) \\ t_L(q_L, q_H) &= \frac{[1 + (1-p) + p\lambda]\theta_H[v(q_L) - v(q_H)] - p(\lambda-1)(\theta_H - \theta_L)v(q_L)}{1+p+(1-p)\lambda} + t_H(q_L, q_H). \end{aligned}$$

Substituting these into (A.3), the first-order conditions are given by

$$\frac{c}{v'(q_L)} + \frac{2\gamma}{p} = \frac{[1+p-p^2+(1-p+p^2)\lambda]\theta_L - (1-p)(\lambda+1)\theta_H}{p[1+p+(1-p)\lambda]} \quad (\text{A.4})$$

$$\frac{c}{v'(q_H)} - \frac{\gamma(\lambda+1)}{1-p} = \frac{[1-2p+2(1+p)\lambda+\lambda^2]\theta_H}{2[1+p+(1-p)\lambda]}. \quad (\text{A.5})$$

Since  $q_L > q_H$  implies  $\frac{c}{v'(q_L)} > \frac{c}{v'(q_H)}$  and also  $\gamma \geq 0$ , the RHS of (A.4) must be greater than the RHS of (A.5), which yields after rearrangement

$$\frac{2-p-2p^2+2(1+p^2)\lambda+p\lambda^2}{2[1+p-p^2+(1-p+p^2)\lambda]} < \frac{\theta_L}{\theta_H} (< 1).$$

This is a contradiction since the LHS of this inequality is bigger than 1 given  $\lambda > 1$ .  $\square$

**Proof of Proposition 4:** We first establish the following lemma:

**Lemma 5.** *Suppose (11) holds. When the optimal reverse-screening menu exists, the qualities must satisfy*

$$\frac{c}{v'(q_L)} + \frac{\mu\theta_L}{p} = \frac{[1+p+(1-p)\lambda](\lambda+1)\theta_L - 2(1-p)(\lambda+1)\theta_H}{2p[1+p+(1-p)\lambda]} \quad (\text{A.6})$$

$$\frac{c}{v'(q_H)} - \frac{\mu\theta_H}{1-p} = \frac{\lambda+1}{1+p+(1-p)\lambda}, \quad (\text{A.7})$$

where  $\mu \geq 0$  is given such that

$$\theta_H v(q_H) - \theta_L v(q_L) \geq 0 \text{ and } \mu[\theta_H v(q_H) - \theta_L v(q_L)] = 0. \quad (\text{A.8})$$

*Proof.* Let us consider the problem of maximizing the seller's profit under the  $(IC''')$  and  $(IR''')$  constraints and the constraints that  $\theta_H v(q_H) - \theta_L v(q_L) \geq 0$  and  $q_L - q_H \geq 0$ . Note that the last constraint must be non-binding for there to exist an optimal reverse-screening menu. Note first that we can ignore  $(IR''_H)$  since it is implied by  $(IC''_H)$  and  $(IR''_L)$  for the same reason as in (A.1). It is also clear that either  $(IC''_H)$  or  $(IC''_L)$  must be binding at the optimum. As in the proof of Lemma 1 above, we let  $t_L(q_L, q_H)$  and  $t_H(q_L, q_H)$  denote the transfers that are obtained by combining two binding constraints,  $(IR''_L)$  and one of the  $(IC'')$  constraints. To check that both  $(IC'')$  constraints are satisfied if one of them is binding, write  $(IC'_H)$  and  $(IC'_L)$  together as

$$(\lambda+1)\theta_L[v(q_L) - v(q_H)] \geq [1+p+(1-p)\lambda](t_L - t_H) \geq 2\theta_H[v(q_L) - v(q_H)].$$

From this, it is clear that if one of the inequalities is satisfied as equality, then the other inequality must be satisfied since  $q_L \geq q_H$  and (11) holds. So we can ignore the  $(IC''')$  constraint that is not set to be binding. Let us now write the Lagrangian for the seller's maximization problem as

$$\begin{aligned} \mathcal{L}(q_L, q_H, \gamma, \mu) = & p(t_L(q_L, q_H) - cq_L) + (1-p)(t_H(q_L, q_H) - cq_H) \\ & + \gamma[q_L - q_H] + \mu[\theta_H v(q_H) - \theta_L v(q_L)]. \end{aligned} \quad (\text{A.9})$$

Since we are looking for the optimal reverse-screening menu,  $\gamma$  is set equal to zero. Depending on which  $(IC''')$  constraint is binding, we consider two cases:

CASE 1: Suppose that  $(IC''_L)$  is binding. Then,

$$t_L(q_L, q_H) = \frac{(\lambda+1)\theta_L[(\lambda-1)(1-p)v(q_H) + 2v(q_L)]}{2[1+p+(1-p)\lambda]}$$

$$t_H(q_L, q_H) = \frac{(\lambda + 1)\theta_L v(q_H)}{2}.$$

Substituting these into (A.9) and applying the first-order conditions yield after rearrangement

$$\frac{c}{v'(q_L)} + \frac{\mu\theta_L}{p} = \frac{(\lambda + 1)\theta_L}{1 + p + (1 - p)\lambda} \quad (\text{A.10})$$

$$\frac{c}{v'(q_H)} - \frac{\mu\theta_H}{1 - p} = \frac{(\lambda + 1)^2\theta_L}{2[1 + p + (1 - p)\lambda]} = \left(\frac{\lambda + 1}{2}\right) \frac{(\lambda + 1)\theta_L}{1 + p + (1 - p)\lambda}. \quad (\text{A.11})$$

Since  $q_L > q_H$  and  $\mu \geq 0$ , the LHS of (A.10) exceeds that of (A.11), which contradicts that the RHS of (A.11) is greater than that of (A.10) given  $\lambda > 1$ .

CASE 2: Suppose that  $(IC_H'')$  is binding. Then,

$$\begin{aligned} t_H(q_L, q_H) &= \frac{(\lambda + 1)\theta_L v(q_L)}{2} - \theta_H[v(q_L) - v(q_H)] \\ t_L(q_L, q_H) &= \frac{(\lambda + 1)\theta_L v(q_L)}{2} - \theta_H \frac{(1 - p)(\lambda - 1)}{1 + p + (1 - p)\lambda} [v(q_L) - v(q_H)]. \end{aligned}$$

Substituting these into (A.9) and applying the first-order condition yield (A.6) and (A.7). Then, (A.8) corresponds to the complementary slackness condition for the constraint  $\theta_H v(q_H) \geq \theta_L v(q_L)$ .  $\square$

Given Lemma 5 and its proof, it is clear that any reverse-screening menu is dominated by the optimal pooling menu if the quality constraint,  $q_L - q_H \geq 0$ , is binding, in which case the other constraint,  $\theta_L v(q_L) - \theta_H v(q_H) \geq 0$ , must be non-binding so  $\mu = 0$ . Then, substituting  $\mu = 0$ , the RHS of (A.7) must be no less than that of (A.6) in order for the quality constraint to be binding, which yields (12) after rearrangement. Now we can rewrite (12) as

$$\lambda \leq \frac{1}{1 - p} \left( \frac{2\theta_H}{\theta_L} - (1 + p) \right) \equiv \lambda^R(p, \theta_H/\theta_L).$$

It is straightforward to check that  $\lambda^R(p, \theta_H/\theta_L)$  increases with both arguments. This completes the proof of Proposition 4.  $\square$

**Proof of Lemma 4:** Note first that the lowest type  $\underline{\theta}$  participates only if

$$\begin{aligned} U(\underline{\theta}) &= V(\underline{\theta}) - t(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} (t(s) - t(\underline{\theta})) dF(s) - \lambda \int_{\underline{\theta}}^{\bar{\theta}} (V(s) - V(\underline{\theta})) dF(s) \\ &\geq \int_{\underline{\theta}}^{\bar{\theta}} t(s) dF(s) - \lambda \int_{\underline{\theta}}^{\bar{\theta}} V(s) dF(s) \end{aligned} \quad (\text{A.12})$$

or

$$V(\underline{\theta})(1 + \lambda) - 2t(\underline{\theta}) \geq 0,$$

which implies that (25) is maximized by setting  $t(\underline{\theta}) = (\frac{1+\lambda}{2})V(\underline{\theta})$  for any  $q(\underline{\theta})$  chosen. Given this, it is easy to verify that the participation constraints for all other types are also satisfied since, from (23),  $U'(\theta') \geq 0$  or the equilibrium payoff  $U(\cdot)$  is increasing while the outside payoff is constant irrespective of type realization, as shown in (A.12).  $\square$

**Proof of Theorem 2:** We first derive the following lemma whose proof can be found in the Supplementary Material:

**Lemma 6.** *A quality schedule that solves [P<sup>c</sup>] must be constant wherever  $J(\cdot, \lambda)$  is decreasing.*

To make use of Lemma 6, it suffices to check that  $J_\theta(\theta, \lambda)$  is negative in the desired range. To do so, let us do some tedious calculation to obtain

$$J_\theta(\theta, \lambda) = J'(\theta)G_\theta(\theta, \lambda) - \frac{1 - F(\theta)}{f(\theta)}G_{\theta\theta}(\theta, \lambda),$$

where

$$G_\theta(\theta, \lambda) = \frac{(\lambda^2 + 2\lambda)[F(\theta)(1 - F(\theta)) - \theta f(\theta)] + 2\lambda + (1 + F(\theta))(2 - F(\theta)) + 3\theta f(\theta)}{(2 - F(\theta) + \lambda F(\theta))^2} \quad (\text{A.13})$$

and

$$G_{\theta\theta}(\theta, \lambda) = \frac{(\lambda - 1)(\lambda + 3)}{(2 - F(\theta) + \lambda F(\theta))^3} \left\{ \lambda[2\theta f(\theta)^2 - F(\theta)(2f(\theta) + \theta f'(\theta))] - 2\theta f(\theta)^2 - (2f(\theta) + \theta f'(\theta))(2 - F(\theta)) \right\}. \quad (\text{A.14})$$

To prove (a), note that  $J_\theta(\bar{\theta}, \lambda) = J'(\bar{\theta})G_\theta(\bar{\theta}, \lambda)$  so the result will obtain if  $G_\theta(\bar{\theta}, \lambda) < 0$  and thus, by continuity,  $J_\theta$  is negative near  $\bar{\theta}$ . It is straightforward to check from (A.13) that  $G_\theta(\bar{\theta}, \lambda) < 0$  is equivalent to requiring  $\frac{\lambda^2 + 2\lambda - 3}{2(\lambda + 1)} > \frac{1}{\theta f(\bar{\theta})}$ . Note that any solution of [P<sup>c</sup>] is guaranteed to satisfy the global incentive compatibility given Assumption 1.

To prove (b), we check that under the stated conditions, (i)  $G_\theta(\theta, \lambda) < 0, \forall \theta$  if  $\lambda > \bar{\lambda}_1$  for some  $\bar{\lambda}_1$ ; (ii)  $G_{\theta\theta}(\theta, \lambda) > 0, \forall \theta$  if  $\lambda > \bar{\lambda}_2$  for some  $\bar{\lambda}_2$ .

For (i), note first that the coefficient for the quadratic term  $\lambda^2$  in the numerator of (A.13) is negative since  $F(\theta)(1 - F(\theta)) \leq F(\theta) < \theta f(\theta)$ , where the inequality is due

to the condition  $\theta f(\theta) > F(\theta)$ . This implies that one can find sufficiently large  $\lambda$ , say  $\bar{\lambda}_1(\theta)$ , such that  $G_\theta(\theta, \lambda) < 0$  if  $\lambda > \bar{\lambda}_1(\theta)$ . Clearly,  $\bar{\lambda}_1(\theta)$  is continuous in  $\theta$  so we can let  $\bar{\lambda}_1 = \max_{\theta \in [\underline{\theta}, \bar{\theta}]} \bar{\lambda}_1(\theta)$ .

For (ii), let us first see that the expression in the square bracket in (A.14) is positive:

$$2\theta f(\theta)^2 - F(\theta)(2f(\theta) + \theta f'(\theta)) \geq 2\theta f(\theta)^2 - 2F(\theta)f(\theta) = 2f(\theta)(\theta f(\theta) - F(\theta)) > 0,$$

where the weak inequality follows from  $f'(\theta) \leq 0$ . Thus, one can find sufficiently large  $\lambda$ , say  $\bar{\lambda}_2(\theta)$ , such that  $G_{\theta\theta}(\theta, \lambda) > 0$  if  $\lambda > \bar{\lambda}_2(\theta)$ . Also,  $\bar{\lambda}_2(\theta)$  is continuous in  $\theta$  so we can let  $\bar{\lambda}_2 = \max_{\theta \in [\underline{\theta}, \bar{\theta}]} \bar{\lambda}_2(\theta)$ .

Letting  $\bar{\lambda} = \max\{\bar{\lambda}_1, \bar{\lambda}_2\}$ ,  $J_\theta(\cdot, \lambda)$  is negative everywhere if  $\lambda > \bar{\lambda}$ . Then, by Lemma 6, the optimal quality schedule must be constant everywhere. This schedule clearly satisfies the global incentive compatibility, which completes the proof.  $\square$

**Proof of Proposition 6:** Given the reverse-screening menu where  $q(\cdot)$  is decreasing and  $V(\cdot)$  is increasing,  $t(\cdot)$  must also be decreasing so we can write the payoff of type  $\theta$  reporting  $\theta'$  as

$$U(\theta'; \theta) = \theta v(q(\theta')) - t(\theta') + \left[ \int_{\underline{\theta}}^{\hat{\theta}(\theta, \theta')} (\theta v(q(\theta')) - V(s)) dF(s) + \int_{\underline{\theta}}^{\theta'} (t(s) - t(\theta')) dF(s) \right] - \lambda \left[ \int_{\hat{\theta}(\theta, \theta')}^{\bar{\theta}} (V(s) - \theta v(q(\theta'))) dF(s) + \int_{\theta'}^{\bar{\theta}} (t(\theta') - t(s)) dF(s) \right]. \quad (\text{A.15})$$

Applying the envelope theorem yields

$$U'(\theta) = v(q(\theta)) [1 + F(\theta) + \lambda(1 - F(\theta))], \quad (\text{A.16})$$

while setting  $\theta' = \theta$  in (A.15) and differentiating it with  $\theta$  yield

$$U'(\theta) = (V'(\theta) - t'(\theta)) [1 + F(\theta) + \lambda(1 - F(\theta))]. \quad (\text{A.17})$$

Equating (A.17) with (A.16) yields

$$t'(\theta) = \theta(v(q(\theta)))'. \quad (\text{A.18})$$

Given this equation, the seller's expected revenue can be expressed as

$$\int_{\underline{\theta}}^{\bar{\theta}} t(s) dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \left( \int_{\underline{\theta}}^{\theta} t'(s) ds + t(\underline{\theta}) \right) dF(\theta)$$

$$\begin{aligned}
&= \int_{\underline{\theta}}^{\bar{\theta}} v(q(\theta)) \left( \theta - \frac{1 - F(\theta)}{f(\theta)} \right) dF(\theta) + t(\underline{\theta}) - V(\underline{\theta}) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} v(q(\theta)) J(\theta) dF(\theta) + t(\underline{\theta}) - V(\underline{\theta}), \tag{A.19}
\end{aligned}$$

where the second equality follows from (A.18) and integration by parts. Next, the participation constraint for the type  $\underline{\theta}$ , can be written as

$$\begin{aligned}
U(\underline{\theta}) &= V(\underline{\theta}) - t(\underline{\theta}) - \lambda \left[ \int_{\underline{\theta}}^{\bar{\theta}} (V(s) - V(\underline{\theta})) dF(s) + \int_{\underline{\theta}}^{\bar{\theta}} (t(\underline{\theta}) - t(s)) dF(s) \right] \\
&\geq \int_{\underline{\theta}}^{\bar{\theta}} t(s) dF(s) - \lambda \int_{\underline{\theta}}^{\bar{\theta}} V(s) dF(s),
\end{aligned}$$

or

$$t(\underline{\theta}) - V(\underline{\theta}) \leq \frac{\lambda - 1}{\lambda + 1} \left( \int_{\underline{\theta}}^{\bar{\theta}} t(s) dF(s) \right). \tag{A.20}$$

Thus, from (A.19) and (A.20), we have

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} t(s) dF(\theta) &\leq \int_{\underline{\theta}}^{\bar{\theta}} v(q(\theta)) J(\theta) dF(\theta) + t(\underline{\theta}) - V(\underline{\theta}) \\
&\leq \int_{\underline{\theta}}^{\bar{\theta}} v(q(\theta)) J(\theta) dF(\theta) + \frac{\lambda - 1}{\lambda + 1} \left( \int_{\underline{\theta}}^{\bar{\theta}} t(s) dF(s) \right),
\end{aligned}$$

or

$$\int_{\underline{\theta}}^{\bar{\theta}} t(s) dF(\theta) \leq \frac{\lambda + 1}{2} \left( \int_{\underline{\theta}}^{\bar{\theta}} v(q(\theta)) J(\theta) dF(\theta) \right).$$

Thus, the seller's profit is bounded above by

$$\int_{\underline{\theta}}^{\bar{\theta}} \left( \frac{\lambda + 1}{2} v(q(\theta)) J(\theta) - cq(\theta) \right) dF(\theta). \tag{A.21}$$

Now we show that this expression is maximized by setting  $q(\cdot)$  constant. To do so, consider any non-increasing  $q(\cdot)$  and let  $\bar{q}$  denote its expected value i.e.  $\bar{q} = \int_{\underline{\theta}}^{\bar{\theta}} q(\theta) dF(\theta)$ . Then, we must have

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} \left( \frac{\lambda + 1}{2} v(q(\theta)) J(\theta) - cq(\theta) \right) dF(\theta) &\leq \left( \frac{\lambda + 1}{2} \right) \left( \int_{\underline{\theta}}^{\bar{\theta}} v(q(\theta)) dF(\theta) \right) \left( \int_{\underline{\theta}}^{\bar{\theta}} J(\theta) dF(\theta) \right) \\
&\quad - \int_{\underline{\theta}}^{\bar{\theta}} cq(\theta) dF(\theta) \\
&\leq \left( \frac{\lambda + 1}{2} \right) v(\bar{q}) \left( \int_{\underline{\theta}}^{\bar{\theta}} J(\theta) dF(\theta) \right) - c\bar{q}
\end{aligned}$$

$$= \int_{\underline{\theta}}^{\bar{\theta}} \left( \frac{\lambda + 1}{2} v(\bar{q}) J(\theta) - c\bar{q} \right) dF(\theta),$$

where the first inequality follows from the fact that  $v(q(\cdot))$  is non-increasing while  $J(\cdot)$  is increasing, and the second from the Jensen's inequality. Thus, (A.21) is maximized by a constant  $q(\cdot)$ .

Given the constant quality  $\bar{q}$  that maximizes (A.21), the upper bound of the seller's profit can be achieved by setting  $t(\theta) = \frac{\lambda+1}{2}\underline{\theta}v(\bar{q}), \forall \theta$  since

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{\lambda + 1}{2} v(\bar{q}) J(\theta) dF(\theta) = \frac{\lambda + 1}{2} \underline{\theta} v(\bar{q}) = \int_{\underline{\theta}}^{\bar{\theta}} t(\theta) dF(\theta),$$

where the first equality follows from the fact that  $\int_{\underline{\theta}}^{\bar{\theta}} J(\theta) dF(\theta) = \underline{\theta}$ . It is straightforward to check that the (IR) constraint is satisfied.  $\square$

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# Supplementary Material

(Not for Publication)

## (IC) and (IR) Constraints

Here, to help the reader, we provide the full expressions of (IC) and (IR) constraints appearing in Section 3.

### Pooling Menu:

$$\begin{aligned} u(r|\theta_L, R) &= \theta_L v(q) - t - (1-p)\lambda(\theta_H - \theta_L)v(q) \\ &\geq u(\emptyset|\theta_L, R) = p[t - \lambda\theta_L v(q)] + (1-p)[t - \lambda\theta_H v(q)], \end{aligned} \quad (IR_L)$$

### Screening Menu:

$$\begin{aligned} u(r_H|\theta_H, R) &= \theta_H v(q_H) - t_H + p[\theta_H v(q_H) - \theta_L v(q_L) - \lambda(t_H - t_L)] \\ &\geq u(r_L|\theta_H, R) = \theta_H v(q_L) - t_L + p(\theta_H - \theta_L)v(q_L) \\ &\quad + (1-p)[(t_H - t_L) - \lambda\theta_H(v(q_H) - v(q_L))] \end{aligned} \quad (IC_H)$$

$$\begin{aligned} u(r_H|\theta_H, R) &= \theta_H v(q_H) - t_H + p[\theta_H v(q_H) - \theta_L v(q_L) - \lambda(t_H - t_L)] \\ &\geq u(\emptyset|\theta_H, R) = p[t_L - \lambda\theta_L v(q_L)] + (1-p)[t_H - \lambda\theta_H v(q_H)] \end{aligned} \quad (IR_H)$$

$$\begin{aligned} u(r_L|\theta_L, R) &= \theta_L v(q_L) - t_L + (1-p)[(t_H - t_L) - \lambda(\theta_H v(q_H) - \theta_L v(q_L))] \\ &\geq u(r_H|\theta_L, R) = \theta_L v(q_H) - t_H - \lambda(1-p)(\theta_H - \theta_L)v(q_H) \\ &\quad + p[\theta_L(v(q_H) - v(q_L)) - \lambda(t_H - t_L)] \end{aligned} \quad (IC_L)$$

$$\begin{aligned} u(r_L|\theta_L, R) &= \theta_L v(q_L) - t_L + (1-p)[(t_H - t_L) - \lambda(\theta_H v(q_H) - \theta_L v(q_L))] \\ &\geq u(\emptyset|\theta_H, R) = p[t_L - \lambda\theta_L v(q_L)] + (1-p)[t_H - \lambda\theta_H v(q_H)] \end{aligned} \quad (IR_L)$$

**Reverse-Screening Menu:** When  $\theta_L v(q_L) > \theta_H v(q_H)$ :

$$\begin{aligned} u(r_H|\theta_H, R) &= \theta_H v(q_H) - t_H + p[(t_L - t_H) - \lambda(\theta_L v(q_L) - \theta_H v(q_H))] \\ &\geq u(r_L|\theta_H, R) = \theta_H v(q_L) - t_L + p(\theta_H - \theta_L)v(q_L) \\ &\quad + (1-p)[\theta_H(v(q_L) - v(q_H)) - \lambda(t_L - t_H)] \end{aligned} \quad (IC_H)$$

$$\begin{aligned} u(r_H|\theta_H, R) &= \theta_H v(q_H) - t_H + p[(t_L - t_H) - \lambda(\theta_L v(q_L) - \theta_H v(q_H))] \\ &\geq u(\emptyset|\theta_H, R) = p[t_L - \lambda\theta_L v(q_L)] + (1-p)[t_H - \lambda\theta_H v(q_H)] \end{aligned} \quad (IR_H)$$

$$\begin{aligned} u(r_L|\theta_L, R) &= \theta_L v(q_L) - t_L + (1-p)[(\theta_L v(q_L) - \theta_H v(q_H)) - \lambda(t_L - t_H)] \\ &\geq u(r_H|\theta_L, R) = \theta_L v(q_H) - t_H - \lambda(1-p)(\theta_H - \theta_L)v(q_H) \\ &\quad + p[(t_L - t_H) - \lambda\theta_L(v(q_L) - v(q_H))] \end{aligned} \quad (IC_L)$$

$$\begin{aligned} u(r_L|\theta_L, R) &= \theta_L v(q_L) - t_L + (1-p)[(\theta_L v(q_L) - \theta_H v(q_H)) - \lambda(t_L - t_H)] \\ &\geq u(\emptyset|\theta_H, R) = p[t_L - \lambda\theta_L v(q_L)] + (1-p)[t_H - \lambda\theta_H v(q_H)]; \end{aligned} \quad (IR_L)$$

When  $\theta_L v(q_L) \leq \theta_H v(q_H)$ ,

$$\begin{aligned} u(r_H|\theta_H, R) &= \theta_H v(q_H) - t_H + p[(t_L - t_H) + (\theta_H v(q_H) - \theta_L v(q_L))] \\ &\geq u(r_L|\theta_H, R) = \theta_H v(q_L) - t_L + p(\theta_H - \theta_L)v(q_L) \\ &\quad + (1-p)[\theta_H(v(q_L) - v(q_H)) - \lambda(t_L - t_H)] \end{aligned} \quad (IC_H)$$

$$\begin{aligned} u(r_H|\theta_H, R) &= \theta_H v(q_H) - t_H + p[(t_L - t_H) + (\theta_H v(q_H) - \theta_L v(q_L))] \\ &\geq u(\emptyset|\theta_H, R) = p[t_L - \lambda\theta_L v(q_L)] + (1-p)[t_H - \lambda\theta_H v(q_H)] \end{aligned} \quad (IR_H)$$

$$\begin{aligned} u(r_L|\theta_L, R) &= \theta_L v(q_L) - t_L - (1-p)\lambda[(t_L - t_H) + (\theta_H v(q_H) - \theta_L v(q_L))] \\ &\geq u(r_H|\theta_L, R) = \theta_L v(q_H) - t_H - \lambda(1-p)(\theta_H - \theta_L)v(q_H) \\ &\quad + p[(t_L - t_H) - \lambda\theta_L(v(q_L) - v(q_H))] \end{aligned} \quad (IC_L)$$

$$\begin{aligned} u(r_L|\theta_L, R) &= \theta_L v(q_L) - t_L - (1-p)\lambda[(t_L - t_H) + (\theta_H v(q_H) - \theta_L v(q_L))] \\ &\geq u(\emptyset|\theta_H, R) = p[t_L - \lambda\theta_L v(q_L)] + (1-p)[t_H - \lambda\theta_H v(q_H)] \end{aligned} \quad (IR_L)$$

## Omitted Proofs

**Proof of Lemma 2:** The proof consists of two claims.

**Claim 1.** *If the optimal menu satisfies  $\theta_H v(q_H) \geq \theta_L v(q_L)$ , then it must be that  $t_H \geq t_L$ .*

*Proof.* Suppose to the contrary that  $t_L > t_H$ . Clearly, we must have both (IR) constraints binding or

$$u(\emptyset|\theta_H, R) = u(r_L|\theta_L, R) = \theta_L v(q_L) - t_L - (1-p)\lambda[\theta_H v(q_H) - \theta_L v(q_L) + t_L - t_H] \quad (\text{SA.1})$$

$$u(\emptyset|\theta_L, R) = u(r_H|\theta_H, R) = \theta_H v(q_H) - t_H + p[\theta_H v(q_H) - \theta_L v(q_L) + t_L - t_H]. \quad (\text{SA.2})$$

Since  $u(\emptyset|\theta_H, R) = u(\emptyset|\theta_L, R)$ , we can equate LHS of (SA.1) and (SA.2) to obtain after rearrangement

$$[1 + p + (1-p)\lambda](t_L - t_H) = [1 + p + (1-p)\lambda](\theta_L v(q_L) - \theta_H v(q_H)),$$

which is a contradiction since  $t_L - t_H > 0$  but  $\theta_L v(q_L) - \theta_H v(q_H) \leq 0$ .  $\square$

**Claim 2.** *It is never optimal to offer a menu with  $\theta_L v(q_L) > \theta_H v(q_H)$ .*

*Proof.* Suppose to the contrary that  $\theta_L v(q_L) > \theta_H v(q_H)$  at the optimum. A similar argument to that in the proof of Claim 1 can be used to show that  $t_L \geq t_H$ . Given this, we can write the (IR) constraints as

$$\begin{aligned} t_H &\leq \frac{(\lambda+1)}{2} \theta_H v(q_H) \\ t_L &\leq t_H + \frac{\theta_L v(q_L) - \theta_H v(q_H)}{B(p, \lambda)}. \end{aligned}$$

Since both constraint must clearly be binding at the optimum, we can substitute these into the objective function and take the first-order conditions as follows:

$$\begin{aligned} \frac{c}{v'(q_L)} &= \frac{\theta_L}{B(p, \lambda)} \\ \frac{c}{v'(q_H)} &= \frac{[(\lambda+1)B(p, \lambda) - 2p] \theta_H}{2(1-p)B(p, \lambda)}. \end{aligned}$$

This, however, yields a contradiction since

$$\frac{c}{v'(q_H)} - \frac{c}{v'(q_L)} = \frac{(\lambda+1)B(p, \lambda)\theta_H - 2[p\theta_H + (1-p)\theta_L]}{2(1-p)B(p, \lambda)} \geq \frac{[(\lambda+1)B(p, \lambda) - 2] \theta_H}{2(1-p)B(p, \lambda)} \geq 0,$$

where the last inequality holds since  $(\lambda+1)B(p, \lambda) \geq (\lambda+1)B(0, \lambda) = 2, \forall \lambda, p$ .  $\square$

Clearly, combining Claim 1 and 2 leads to the desired result.  $\square$

**Proof of Proposition 5:** The proof will be done if the optimal screening menu that solves  $[P']$  is shown to violate the monotonicity constraint,  $q_H > q_L$ , whenever (10) in the main text holds.

To first identify possible sets of binding constraints for  $[P']$ , we rely on a graphical illustration for simplicity.<sup>27</sup> First of all, with fixed qualities  $q_L$  and  $q_H > q_L$ , the green area in Figure 5 depicts the set of transfers where  $(IC_H)$ ,  $(IC_L)$ , and  $(IR_L)$  are satisfied. (The line labeled  $IC_H$ , for instance, is where  $(IC_H)$  is binding.) Without  $(EA)$ , the optimal transfer scheme would be represented by a point  $A$  since the red line and its parallels correspond to the seller's *iso-revenue* lines.

Now let the blue line in Figure 5 denote the set of transfers where  $(EA)$  is binding. Then,  $(EA)$  is satisfied below this line.<sup>28</sup> Note that if the blue line stays above  $A$  given the optimal quality level  $(q_L^s, q_H^s)$  in Proposition 3, then  $(EA)$  is non-binding so the optimal screening menu described in Section 3.2 must solve  $[P']$  as well, which implies that screening is dominated by pooling if (10) holds.

Turning to the case in which  $(EA)$  is binding, we can consider four possibilities for the solution of  $[P']$  where the following sets of constraints are binding: in addition to  $(EA)$ , (a)  $(IC_H)$  and  $(IR_L)$  are binding; (b)  $(IR_L)$  is binding; (c)  $(IC_L)$  and  $(IR_L)$  are binding; or (d)  $(IC_L)$  is binding.

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<sup>27</sup>A formal proof is a straightforward translation of graphical illustration and is thus omitted. Note that  $(IR_H)$  constraint, which is implied by these constraints, can be ignored.

<sup>28</sup>Though the graph here is only drawn in case it is negatively sloped (i.e.  $1 - p(\lambda - 1) > 0$ ), the subsequent analysis is not changing at all in case it is positively sloped since the slope is always between  $-p/(1 - p)$  and 1.

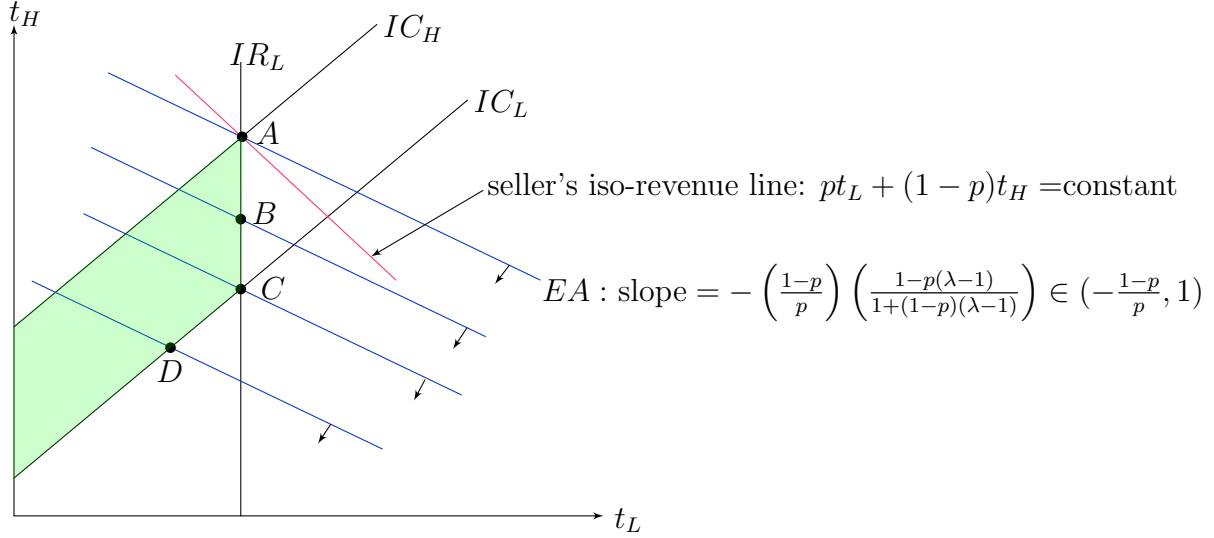


Figure 5

These four cases are shown in Figure 5 via the points,  $A$ ,  $B$ ,  $C$ , and  $D$ , and the corresponding iso-revenue lines passing through them. It is clear that in these cases, the seller's revenue cannot be increased by adjusting the level of transfers only. Since we have found no simple way to further determine which case(s) must hold at the optimum, we go through all four cases and establish the desired result in each case. We only provide the proof for the case (a) and (d) since an analogous argument can be used to deal with the other cases. Before giving the proof, let us first rewrite the buyer's ex-ante utility  $U(R)$  defined in (18) as

$$\begin{aligned}
 U(R) &= p(\theta_L v(q_L) - t_L) + (1-p)(\theta_H v(q_H) - t_H) \\
 &\quad - p(1-p)(\lambda-1)(\theta_H v(q_H) - \theta_L v(q_L) + t_H - t_L). \tag{SA.3}
 \end{aligned}$$

**Claim 1.** *Suppose that the case (a) holds at the solution of  $[P']$ . Then, pooling dominates screening if (10) holds.*

*Proof.* In the proof of this and next claims, From now, we abbreviate  $B(p, \theta_H/\theta_L)$  (defined in (6)) to  $B$  for simplicity. Set up a Lagrangian for the problem  $[P']$ :

$$\begin{aligned}
 \mathcal{L}(R, \gamma, \mu, \tau) &= p(t_L - cq_L) + (1-p)(t_H - cq_H) \\
 &\quad + \gamma \left[ t_L - \frac{\lambda+1}{2} \theta_L v(q_L) \right] + \mu \left[ t_L - t_H + \frac{\theta_H [v(q_H) - v(q_L)]}{B} \right] + \tau U(R).
 \end{aligned}$$

Note that  $\gamma$ ,  $\mu$ , and  $\tau$  are nonnegative multipliers for  $(IR_L)$ ,  $(IC_H)$ , and  $(EA)$ , respectively.<sup>29</sup>

Taking the first-order condition with respect to  $t_H$  yields

$$(1-p) - \mu - \tau[1-p+p(1-p)(\lambda-1)] = 0,$$

which implies that  $\tau = \frac{1-p-\mu}{1-p+p(1-p)(\lambda-1)} < 1$ . Now using the binding  $(IR_L)$  and  $(IC_H)$  constraints, one can obtain

$$pt_L + (1-p)t_H = \frac{\lambda+1}{2}\theta_L v(q_L) + (1-p)\frac{\theta_H[v(q_H) - v(q_L)]}{B}.$$

Substituting this and the binding  $(IC_H)$  constraint into the Lagrangian gives after rearrangement:

$$\begin{aligned} \mathcal{L}(q_L, q_H, \tau) = & (1-\tau) \left[ \frac{\lambda+1}{2}\theta_L v(q_L) + (1-p)\frac{\theta_H[v(q_H) - v(q_L)]}{B} \right] - pcq_L - (1-p)cq_H \\ & + \tau \left[ p\theta_L v(q_L) + (1-p)\theta_H v(q_H) - p(1-p)(\lambda-1) \left( \theta_H v(q_H) - \theta_L v(q_L) + \frac{\theta_H[v(q_H) - v(q_L)]}{B} \right) \right]. \end{aligned}$$

Then, the first-order condition w.r.t  $q_L$  and  $q_H$  results in

$$(1-\tau) \underbrace{\left[ \frac{\lambda+1}{2p}\theta_L - \left( \frac{1-p}{p} \right) \frac{\theta_H}{B} \right]}_{\Omega_L} + \tau \underbrace{\left[ \theta_L + (1-p)(\lambda-1) \left( \theta_L + \frac{\theta_H}{B} \right) \right]}_{\Phi_L} = \frac{c}{v'(q_L)} \quad (\text{SA.4})$$

$$(1-\tau) \underbrace{\left[ \frac{\theta_H}{B} \right]}_{\Omega_H} + \tau \underbrace{\left[ \theta_H - p(\lambda-1) \left( \theta_H + \frac{\theta_H}{B} \right) \right]}_{\Phi_L} = \frac{c}{v'(q_H)}. \quad (\text{SA.5})$$

We show that if (10) holds, then the LHS of (SA.4) is no smaller than that of (SA.5) so the constraint  $q_H \geq q_L$  must be binding, implying that pooling dominates screening. For this, it suffices to establish that  $\Omega_L \geq \Omega_H$  and  $\Phi_L \geq \Phi_H$  if (10) holds, since  $\tau > 0$  and  $1-\tau > 0$ . First,  $\Omega_L$  being no smaller than  $\Omega_H$  is equivalent to (10), as can be easily checked. We now establish that  $\Phi_L \geq \Phi_H$  if (10) holds. Rearrange the inequality  $\Phi_L \geq \Phi_H$  to obtain

$$\frac{\theta_L}{\theta_H} \geq \frac{1-p(\lambda-1) - \frac{\lambda-1}{B}}{1+(1-p)(\lambda-1)} = \frac{[1-p(\lambda-1)]B - (\lambda-1)}{[1+(1-p)(\lambda-1)]B}. \quad (\text{SA.6})$$

Having (10) rewritten as  $\theta_L/\theta_H \geq \frac{2B}{\lambda+1}$ , (SA.6) will hold if it holds that  $\frac{2B}{\lambda+1} \geq \frac{[1-p(\lambda-1)]B - (\lambda-1)}{[1+(1-p)(\lambda-1)]B}$ , which is true since one can obtain after some rearrangement using the definition of  $B$ ,

$$\frac{2}{(\lambda+1)B} - \frac{[1-p(\lambda-1)]B - (\lambda-1)}{[1+(1-p)(\lambda-1)]B} = \frac{(\lambda-1)(\lambda+3)[(1-p+p^2)\lambda+1-p^2]}{(\lambda+1)(p\lambda+2-p)[(1-p)\lambda+p]} > 0,$$

completing the proof of this claim.  $\square$

<sup>29</sup>The specified form of  $(IR_L)$  and  $(IC_H)$  here follows from the inequality version of (4) and (7).

**Claim 2.** Suppose that the case (d) holds at the solution of  $[P']$ . Then, pooling dominates screening if (10) holds.

*Proof.* Using the binding (EA) constraint, letting  $U(R)$  in (SA.3) be equal to zero yields

$$\begin{aligned} & pt_L + (1-p)t_H \\ &= p\theta_L v(q_L) + (1-p)\theta_H v(q_H) - p(1-p)(\lambda-1)(\theta_H v(q_H) - \theta_L v(q_L) + t_H - t_L) \\ &= p\theta_L v(q_L) + (1-p)\theta_H v(q_H) - p(1-p)(\lambda-1) \left( \theta_H v(q_H) - \theta_L v(q_L) + \frac{\theta_L[v(q_H) - v(q_L)]}{B} \right), \end{aligned}$$

where the second equality follows from substituting the binding (IC<sub>L</sub>) constraint,  $t_H - t_L = \frac{\theta_L[v(q_H) - v(q_L)]}{B}$ . Then, the seller's profit, which is equal to the above expression minus the expected cost (i.e.  $pcq_L + (1-p)cq_H$ ), has now been expressed in terms of the two qualities only. We take the first-order conditions w.r.t  $q_L$  and  $q_H$  and obtain after rearrangement

$$\theta_L + (1-p)(\lambda-1) \left( \theta_L + \frac{\theta_L}{B} \right) = \frac{c}{v'(q_L)} \quad (\text{SA.7})$$

$$\theta_H - p(\lambda-1) \left( \theta_H + \frac{\theta_L}{B} \right) = \frac{c}{v'(q_H)}. \quad (\text{SA.8})$$

Again we show that the LHS of (SA.7) is no smaller than that of (SA.8), which can be rewritten as

$$\frac{\theta_L}{\theta_H} \geq \frac{[1-p(\lambda-1)]B}{[1+(1-p)(\lambda-1)]B + (\lambda-1)}. \quad (\text{SA.9})$$

We show that (10) implies (SA.9) by establishing  $\frac{2B}{\lambda+1} \geq \frac{[1-p(\lambda-1)]B}{[1+(1-p)(\lambda-1)]B + (\lambda-1)}$ , which holds true since

$$\begin{aligned} & \frac{2B}{\lambda+1} - \frac{[1-p(\lambda-1)]B}{[1+(1-p)(\lambda-1)]B + (\lambda-1)} \\ &= \frac{(\lambda-1)(\lambda+3)[p^3\lambda^2 + (2-2p+p^2-2p^3)\lambda + 2-p^2+p^3]}{(\lambda+1)(p\lambda+2-p)[(1-p^2)\lambda^2 + (2-p+2p^2)\lambda - 1 + p - p^2]} > 0. \end{aligned} \quad ^{30}$$

□

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<sup>30</sup>This inequality can be checked by observing

$$\begin{aligned} p^3\lambda^2 + (2-2p+p^2-2p^3)\lambda + 2-p^2+p^3 &\geq p^3\lambda + (2-2p+p^2-2p^3)\lambda + 2-p^2+p^3 \\ &= (2-2p+p^2-p^3)\lambda + 2-p^2+p^3 > 0 \end{aligned}$$

Before wrapping up the proof of Proposition 5, we note that the area in Figure 3 where pooling dominates screening in the problem  $[P']$  is obtained by finding the values of  $(\lambda, p)$  for which both conditions (SA.6) and (SA.9) are satisfied. We also note that the condition for pooling to solve  $[P']$  in case (b) or (c) holds is precisely the same as (SA.6) or (SA.9).  $\square$

**Proof of Lemma 3:** To simplify notation, let  $\hat{\theta}$  denote  $\hat{\theta}(\theta, \theta')$ . It suffices to prove that  $\frac{\partial U(\theta'; \theta)}{\partial \theta'} \geq (\leq) 0$  if  $\theta' \leq (\geq) \theta$ . To that end, differentiate  $U(\theta'; \theta)$  with  $\theta'$  to obtain

$$\begin{aligned} \frac{\partial U(\theta'; \theta)}{\partial \theta'} &= \theta (v(q(\theta')))' \left[ 1 + F(\hat{\theta}) + \lambda(1 - F(\hat{\theta})) \right] - t'(\theta') [2 - F(\theta') + \lambda F(\theta')] \\ &= (v(q(\theta')))' \left[ \theta \left( 1 + F(\hat{\theta}) + \lambda(1 - F(\hat{\theta})) \right) - \theta' (1 + F(\theta') + \lambda(1 - F(\theta'))) \right], \end{aligned} \tag{SA.10}$$

where the second equality follows from substituting (24). For the optimality of  $\theta' = \theta$ , it suffices to show that the expression in (SA.10) is nonnegative (nonpositive) if  $\theta' < (>) \theta$ . If  $\theta' < \theta$ , then  $\theta' \leq \hat{\theta} \leq \theta$  and, due to Assumption 1,

$$\theta \left( 1 + F(\hat{\theta}) + \lambda(1 - F(\hat{\theta})) \right) \geq \hat{\theta} \left( 1 + F(\hat{\theta}) + \lambda(1 - F(\hat{\theta})) \right) \geq \theta' (1 + F(\theta') + \lambda(1 - F(\theta')))$$

implying that (SA.10) is nonnegative. Also, if  $\theta' > \theta$ , then  $\theta' \geq \hat{\theta} \geq \theta$  and, thus,

$$\theta \left( 1 + F(\hat{\theta}) + \lambda(1 - F(\hat{\theta})) \right) \leq \hat{\theta} \left( 1 + F(\hat{\theta}) + \lambda(1 - F(\hat{\theta})) \right) \leq \theta' (1 + F(\theta') + \lambda(1 - F(\theta'))),$$

implying that (SA.10) is nonpositive, as desired.  $\square$

**Proof of Lemma 6:** Following Myerson (1981) or Toikka (2009), let us first define the ironed-out function of  $J(\cdot, \lambda)$  as follows: Let

$$H(\theta, \lambda) := \int_{\theta}^{\theta} J(s, \lambda) ds,$$

$G(\cdot, \lambda)$  be the convex hull of  $H(\cdot, \lambda)$  or the highest convex function such that  $G(\theta, \lambda) \leq H(\theta, \lambda), \forall \theta$ , and

$$\bar{J}(\theta, \lambda) := \frac{\partial G(\theta, \lambda)}{\partial \theta}.$$

and

$$\begin{aligned} (1 - p^2)\lambda^2 + (2 - p + 2p^2)\lambda - 1 + p - p^2 &\geq (1 - p^2)\lambda^2 + (2 - p + 2p^2) - 1 + p - p^2 \\ &= (1 - p^2)\lambda^2 + 1 + p^2 > 0. \end{aligned}$$

Letting  $\mathcal{Q}$  denote the set of nondecreasing quality schedules, Toikka (2009) shows that

$$\sup_{q(\cdot) \in \mathcal{Q}} \int_{\theta}^{\bar{\theta}} J(\theta, \lambda) q(\theta) d\theta = \sup_{q(\cdot)} \int_{\theta}^{\bar{\theta}} \bar{J}(\theta, \lambda) q(\theta) d\theta.$$

Note that the RHS is an unconstrained optimization problem. With  $G$  being the convex hull of  $H$ , we must have  $G$  being linear in  $\theta$  in the range where  $\frac{\partial H}{\partial \theta} = J(\cdot, \lambda)$  is decreasing, which implies that  $\bar{J}(\cdot, \lambda)$  is constant in that range. This implies that the optimal quality schedule solving the above unconstrained problem, and thus  $[P^c]$ , must be constant where  $J(\cdot, \lambda)$  is decreasing.  $\square$

## An Alternative Specification of Gain-Loss Utility

We show that if gain-loss utility is measured in one dimension, i.e. by the gross utility, the monopolist's problem is no different from the standard one.

Let us begin with  $(IC_H)$ . Depending on the relative sizes of intrinsic utility, we have the following three cases for  $(IC_H)$  constraint:

1. When  $\theta_H v(q_H) - t_H \geq \theta_H v(q_L) - t_L$ ,  $(IC_H)$  is written as

$$\begin{aligned} & \theta_H v(q_H) - t_H + p[\theta_H v(q_H) - t_H - (\theta_L v(q_L) - t_L)] \\ & \geq \theta_H v(q_L) - t_L + p(\theta_H - \theta_L)v(q_L) - (1-p)\lambda[\theta_H v(q_H) - t_H - (\theta_H v(q_L) - t_L)], \end{aligned}$$

which reduces to

$$[\theta_H v(q_H) - t_H - (\theta_H v(q_L) - t_L)][1 + p + (1-p)\lambda] \geq 0$$

which is exactly the same with the standard  $(IC_H)$ .

2. When  $\theta_H v(q_H) - t_H < \theta_L v(q_L) - t_L$ ,  $(IC_H)$  becomes

$$\begin{aligned} & \theta_H v(q_H) - t_H - p\lambda[\theta_H v(q_H) - t_H - (\theta_L v(q_L) - t_L)] \\ & \geq \theta_H v(q_L) - t_L + p(\theta_H - \theta_L)v(q_L) + (1-p)[\theta_H v(q_L) - t_L - (\theta_H v(q_H) - t_H)], \end{aligned}$$

which cannot be satisfied since the intrinsic utility of LHS is smaller than that of RHS by assumption, and the gain-loss utility of LHS is negative while that of RHS is positive.

3. When  $\theta_L v(q_L) - t_L \leq \theta_H v(q_H) - t_H < \theta_H v(q_L) - t_L$ , ( $IC_H$ ) is then

$$\begin{aligned} & \theta_H v(q_H) - t_H + p[\theta_H v(q_H) - t_H - (\theta_L v(q_L) - t_L)] \\ & \geq \theta_H v(q_L) - t_L + p(\theta_H - \theta_L)v(q_L) + (1-p)[\theta_H v(q_L) - t_L - (\theta_H v(q_H) - t_H)], \end{aligned}$$

which can be rearranged to

$$[\theta_H v(q_H) - t_H - (\theta_H v(q_L) - t_L)][1 + p + (1-p)] \geq 0,$$

contradicting with the assumption.

From this, we can focus on the case  $\theta_H v(q_H) - t_H \geq \theta_H v(q_L) - t_L$  for ( $IC_H$ ), which implies  $\theta_H v(q_H) - t_H > \theta_L v(q_L) - t_L$ . So ( $IR_H$ ) can be ignored since  $u(r_H|\theta_H, R) > u(r_L|\theta_L, R) \geq u(\emptyset|\theta_L, R) = u(\emptyset|\theta_H, R)$ , where the second inequality follows from ( $IR_L$ ).

As for ( $IR_L$ ), we also have three cases to consider:

1. When  $\theta_H v(q_H) - t_H < 0$ , ( $IR_L$ ) is written as

$$\begin{aligned} & \theta_L v(q_L) - t_L - (1-p)\lambda[\theta_H v(q_H) - t_H - (\theta_L v(q_L) - t_L)] \\ & \geq p[t_L - \theta_L v(q_L)] + (1-p)[t_H - \theta_H v(q_H)] \end{aligned}$$

which reduces to

$$[\theta_L v(q_L) - t_L][1 + p + (1-p)\lambda] \geq [\theta_H v(q_H) - t_H](1-p)(\lambda - 1),$$

contradicting with the assumption  $\theta_L v(q_L) - t_L < \theta_H v(q_H) - t_H < 0$ .

2. When  $\theta_L v(q_L) - t_L \geq 0$ , ( $IR_L$ ) is written as

$$\begin{aligned} & \theta_L v(q_L) - t_L - (1-p)\lambda[\theta_H v(q_H) - t_H - (\theta_L v(q_L) - t_L)] \\ & \geq -p\lambda[\theta_L v(q_L) - t_L] - (1-p)\lambda[\theta_H v(q_H) - t_H], \end{aligned}$$

which implies  $[\theta_L v(q_L) - t_L](1 + \lambda) \geq 0$ .

3. When  $\theta_L v(q_L) - t_L < 0 \leq \theta_H v(q_H) - t_H$ , ( $IR_L$ ) is written as

$$\begin{aligned} & \theta_L v(q_L) - t_L - (1-p)\lambda[\theta_H v(q_H) - t_H - (\theta_L v(q_L) - t_L)] \\ & \geq p[t_L - \theta_L v(q_L)] - (1-p)\lambda[\theta_H v(q_H) - t_H], \end{aligned}$$

which implies  $[\theta_L v(q_L) - t_L](1-p)(\lambda - 1) \geq 0$ .

Thus,  $(IR_L)$  also boils down to be the same as in the standard model.

Lastly, let us consider  $(IC_L)$ . Because we know that  $\theta_H v(q_H) - t_H > \theta_L v(q_L) - t_L$ , there are only two cases to be considered,  $\theta_L v(q_L) - t_L \geq \theta_L v(q_H) - t_H$  or not. When the inequality is true,  $(IC_L)$  is written as

$$\begin{aligned} & \theta_L v(q_L) - t_L - (1-p)\lambda[\theta_H v(q_H) - t_H - (\theta_L v(q_L) - t_L)] \\ & \geq \theta_L v(q_H) - t_H - (1-p)\lambda(\theta_H - \theta_L)v(q_H) - p\lambda[\theta_L v(q_L) - t_L - (\theta_L v(q_H) - t_H)] \end{aligned}$$

which reduces to

$$[\theta_L v(q_L) - t_L - (\theta_L v(q_H) - t_H)][1 + \lambda] \geq 0, \tag{SA.11}$$

yielding the same  $(IC_L)$  as in the standard model. When the inequality is not true, the resulting  $(IC_L)$  is the same as (SA.11) except that the multiplier of LHS becomes  $1 + p + (1-p)\lambda$  instead of  $1 + \lambda$ .