Photometric Stereo
Photometric Stereo v.s. Structure from Shading [1]

- **Photometric stereo** is a technique in computer vision for estimating the *surface normals* of objects by observing that object under different lighting conditions. The technique was originally introduced by Woodham in 1980.


- The special case where the data is a *single image* is known as *shape from shading*, and was analyzed by B. K. P. Horn in 1989.

## Radiometry Units [7]

### SI radiometry units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>SI unit</th>
<th>Abbr.</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiant energy</td>
<td>Q</td>
<td>joule</td>
<td>J</td>
<td>energy</td>
</tr>
<tr>
<td>Radiant flux</td>
<td>( \Phi )</td>
<td>watt</td>
<td>W</td>
<td>radiant energy per unit time, also called radiant power</td>
</tr>
<tr>
<td>Radiant intensity</td>
<td>( I )</td>
<td>watt per steradian</td>
<td>( \text{W} \cdot \text{sr}^{-1} )</td>
<td>power per unit solid angle</td>
</tr>
<tr>
<td>Radiance</td>
<td>( L )</td>
<td>watt per steradian per square metre</td>
<td>( \text{W} \cdot \text{sr}^{-1} \cdot \text{m}^{-2} )</td>
<td>power per unit solid angle per unit projected source area. Called intensity in some other fields of study.</td>
</tr>
<tr>
<td>Irradiance</td>
<td>( E, I )</td>
<td>watt per square metre</td>
<td>( \text{W} \cdot \text{m}^{-2} )</td>
<td>power incident on a surface. Sometimes confusingly called “intensity”.</td>
</tr>
<tr>
<td>Radiant exitance / Radiant emittance</td>
<td>( M )</td>
<td>watt per square metre</td>
<td>( \text{W} \cdot \text{m}^{-2} )</td>
<td>power emitted from a surface.</td>
</tr>
<tr>
<td>Radiosity</td>
<td>( J ) or ( J_{\text{\lambda}} )</td>
<td>watt per square metre</td>
<td>( \text{W} \cdot \text{m}^{-2} )</td>
<td>emitted plus reflected power leaving a surface</td>
</tr>
<tr>
<td>Spectral radiance</td>
<td>( L_{\lambda} ) or ( L_{v} )</td>
<td>watt per steradian per ( \text{m}^{-3} ) or watt per steradian per square metre per hertz</td>
<td>( \text{W} \cdot \text{sr}^{-1} \cdot \text{m}^{-3} ) or ( \text{W} \cdot \text{sr}^{-1} \cdot \text{m}^{-2} \cdot \text{Hz}^{-1} )</td>
<td>Commonly measured in ( \text{W} \cdot \text{sr}^{-1} \cdot \text{m}^{-2} \cdot \text{nm}^{-1} )</td>
</tr>
<tr>
<td>Spectral irradiance</td>
<td>( E_{\lambda} ) or ( E_{v} )</td>
<td>watt per ( \text{m}^{-3} ) or watt per square metre per hertz</td>
<td>( \text{W} \cdot \text{m}^{-3} ) or ( \text{W} \cdot \text{m}^{-2} \cdot \text{Hz}^{-1} )</td>
<td>Commonly measured in ( \text{W} \cdot \text{m}^{-2} \cdot \text{nm}^{-1} )</td>
</tr>
</tbody>
</table>
Radiometry Units

Steradian

A steradian is used to measure "solid" angles

A steradian is related to the surface area of a sphere in the same way a radian is related to the circumference of a circle:

A Radian "cuts out" a length of a circle's circumference equal to the radius.

A Steradian "cuts out" an area of a sphere equal to (radius)$^2$.

http://www.mathsisfun.com/geometry/steradian.html

E-mail: hogijung@hanyang.ac.kr
http://web.yonsei.ac.kr/hgjung
Radiometry Units

Radiant Intensity

Radiant intensity (how brightly something shines) can be measured in watts per steradian (W/sr).

**Example:** You measure the light coming from a powerful globe. Your sensor is 50mm × 50mm in size, and if you hold it 2m away it measures 0.1 Watts. What is the radiant intensity in W/sr?

**Answer:** At 2m, one steradian would cut through $2 \times 2 = 4 \text{ m}^2$ of the sphere.

And because the sensor is relatively small, its flat surface area will be approximately the area of sphere that it occupies. So $0.05 \times 0.05 = 0.0025 \text{m}^2$.

So, one steradian would receive $0.1 \text{ W} \times (4\text{m}^2/0.0025\text{m}^2) = 160 \text{ W/sr}$. 

http://www.mathsisfun.com/geometry/steradian.html

E-mail: hogijung@hanyang.ac.kr
http://web.yonsei.ac.kr/hgjung
Radiometry Units [7]

http://www.light-measurement.com/basic-radiometric-quantities/

Radiant energy

Per unit time

Radiant power

Per unit solid angle

Radiant intensity

Per unit projected-area

Irradiance
The amount of radiant power impinging upon a surface per unit area.

Radiance

Per unit area

Radiant exitance
Radiant emittance

+ reflected light

Radiosity

Fig. 1. Geometry of the radiance definition: $d\phi$ - radiant flux which propagates within an elementary solid angle, $d\omega$, about the direction of observation, $\hat{\zeta}$; $dA$ - elementary area of the radiation source with its normal, $n$, $dA_\parallel = dA \cos\theta$ - projection of the elementary area onto a plane perpendicular to the direction of observation, $\hat{\zeta}$.

http://www.tpdsci.com/tpc/RdmRadERGeoFig.php

E-mail: hogijung@hanyang.ac.kr
http://web.yonsei.ac.kr/hgjung
Photometry is the science of the measurement of light, in terms of its perceived brightness to the human eye. It is distinct from radiometry, which is the science of measurement of radiant energy (including light) in terms of absolute power; rather, in photometry, the radiant power at each wavelength is weighted by a luminosity function (a.k.a. visual sensitivity function) that models human brightness sensitivity.

### SI photometry units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>SI unit</th>
<th>Abbr.</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminous energy</td>
<td>$Q_v$</td>
<td>lumen second</td>
<td>lm·s</td>
<td>units are sometimes called talbots</td>
</tr>
<tr>
<td>Luminous flux</td>
<td>$F$</td>
<td>lumen (= cd·sr)</td>
<td>lm</td>
<td>also called <em>luminous power</em></td>
</tr>
<tr>
<td>Luminous intensity</td>
<td>$I_v$</td>
<td>candela (= lm/sr)</td>
<td>cd</td>
<td>an SI base unit</td>
</tr>
<tr>
<td>Luminance</td>
<td>$L_v$</td>
<td>candela per square metre</td>
<td>cd/m²</td>
<td>units are sometimes called &quot;nits&quot;</td>
</tr>
<tr>
<td>Illuminance</td>
<td>$E_v$</td>
<td>lux (= lm/m²)</td>
<td>lx</td>
<td>Used for light incident on a surface</td>
</tr>
<tr>
<td>Luminous emittance</td>
<td>$M_v$</td>
<td>lux (= lm/m²)</td>
<td>lx</td>
<td>Used for light emitted from a surface</td>
</tr>
<tr>
<td>Luminous efficacy</td>
<td></td>
<td>lumen per watt</td>
<td>lm/W</td>
<td>ratio of luminous flux to radiant flux</td>
</tr>
</tbody>
</table>

E-mail: hogijung@hanyang.ac.kr  
http://web.yonsei.ac.kr/hgjung
Lambert's cosine law [9]

In optics, Lambert's cosine law says that the radiant intensity observed from a "Lambertian" surface is directly proportional to the cosine of the angle $\theta$ between the observer's line of sight and the surface normal.

An important consequence of Lambert's cosine law is that when such a surface is viewed from any angle, it has the same apparent radiance. This means, for example, that to the human eye it has the same apparent brightness (or luminance). It has the same radiance because, although the emitted power from a given area ($dA$) element is reduced by the cosine of the emission angle, the size of the observed area ($dA_{\text{normal}}$) is decreased by a corresponding amount.
Lambertian scatters [9]

When an area element is radiating as a result of being illuminated by an external source, the irradiance (energy or photons/time/area) landing on that area element will be proportional to the cosine of the angle between the illuminating source and the normal.

A Lambertian scatterer will then scatter this light according to the same cosine law as a Lambertian emitter.

This means that although the radiance of the surface depends on the angle from the normal to the illuminating source, it will not depend on the angle from the normal to the observer.
Lambert’s cosine law [9]

Emission rate (photons/s) in a normal and off-normal direction. The number of photons/sec directed into any wedge is proportional to the area of the wedge.
**Lambert’s cosine law [9]**

The observer directly above the area element will be seeing the scene through an aperture of area \(dA_0\) and the area element \(dA\) will subtend a (solid) angle of \(d\Omega_0\). We can assume without loss of generality that the aperture happens to subtend solid angle \(d\Omega\) when "viewed" from the emitting area element. This normal observer will then be recording \(I \, d\Omega \, dA\) photons per second and so will be measuring a radiance of

\[
I_0 = \frac{I \, d\Omega \, dA}{d\Omega_0 \, dA_0} \quad \text{photons/(s·cm}^2\cdot\text{sr}).
\]

The observer at angle \(\theta\) to the normal will be seeing the scene through the same aperture of area \(dA_0\) and the area element \(dA\) will subtend a (solid) angle of \(d\Omega_0 \cos(\theta)\). This observer will be recording \(I \cos(\theta) \, d\Omega \, dA\) photons per second, and so will be measuring a radiance of

\[
I_0 = \frac{I \cos(\theta) \, d\Omega \, dA}{d\Omega_0 \cos(\theta) \, dA_0} = \frac{I \, d\Omega \, dA}{d\Omega_0 \, dA_0} \quad \text{photons/(s·cm}^2\cdot\text{sr}),
\]

which is the same as the normal observer.
The Bidirectional Reflection Distribution Function

- Given an incoming ray \((\theta_i, \phi_i)\) and outgoing ray \((\theta_e, \phi_e)\), what proportion of the incoming light is reflected along outgoing ray?

Answer given by the BRDF: \(\rho(\theta_i, \phi_i, \theta_e, \phi_e)\)
Constraints on the BRDF [5]

Energy conservation

• Quantity of outgoing light $\leq$ quantity of incident light
  – integral of BRDF $\leq 1$

Helmholtz reciprocity

• reversing the path of light produces the same reflectance
Diffuse reflection

- Dull, matte surfaces like chalk or latex paint
- Microfacets scatter incoming light randomly
- Effect is that light is reflected equally in all directions
Diffuse Reflection [5]

Diffuse reflection governed by Lambert’s law

- Viewed brightness does not depend on viewing direction
- Brightness does depend on direction of illumination
- This is the model most often used in computer vision

Lambert’s Law: \( I_e = k_d N \cdot L I_i \)

\( k_d \) is called albedo

BRDF for Lambertian surface

\[ \rho(\theta_i, \phi_i, \theta_e, \phi_e) = k_d \cos \theta_i \]

- \( L, N, V \) unit vectors
- \( I_e = \) outgoing radiance
- \( I_i = \) incoming radiance

E-mail: hogijung@hanyang.ac.kr
http://web.yonsei.ac.kr/hgjung
Specular Reflection [5]

For a perfect mirror, light is reflected about $\mathbf{N}$

$$I_e = \begin{cases} I_i & \text{if } \mathbf{V} = \mathbf{R} \\ 0 & \text{otherwise} \end{cases}$$

Near-perfect mirrors have a **highlight** around $\mathbf{R}$

- common model: $I_e = k_s (\mathbf{V} \cdot \mathbf{R})^{n_s} I_i$

E–mail: hogijung@hanyang.ac.kr
http://web.yonsei.ac.kr/hgjung
Diffuse Reflection and Lambertian BRDF [2]

- Surface appears equally bright from ALL directions! (independent of $\mathbf{V}$)

- Lambertian BRDF is simply a constant: $f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi} \cos \theta_i$

- Surface Radiance: $L = \frac{\rho_d}{\pi} I \cos \theta_i = \frac{\rho_d}{\pi} I \mathbf{n} \cdot \mathbf{s}$

- Commonly used in Vision and Graphics!
Specular Reflection and Mirror BRDF [2]

- Valid for very smooth surfaces.
- All incident light energy reflected in a SINGLE direction (only when $\vec{V} = \vec{r}$).
- Mirror BRDF is simply a double-delta function:

$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s \delta(\theta_i - \theta_v) \delta(\phi_i + \pi - \phi_v)$$

- Surface Radiance:

$$L = I \rho_s \delta(\theta_i - \theta_v) \delta(\phi_i + \pi - \phi_v)$$
Combining Specular and Diffuse [2]

Observed Image Color = a \times \text{Body Color} + b \times \text{Specular Reflection Color}

Does not specify any specific model for Diffuse/specular reflection
Combining Specular and Diffuse [2]

diffuse
specular
diffuse+specular
BRDF Models [5]

• Phenomenological
  – Phong
  – Ward
  – Lafortune et al.
  – Ashikhmin et al.

• Physical
  – Cook–Torrance
  – Dichromatic
  – He et al.

• Here we’re listing only some well-known examples
Phong Illumination Model [5]

Phong approximation of surface reflectance
- Assume reflectance is modeled by three components
  - Diffuse term
  - Specular term
  - Ambient term (to compensate for inter-reflected light)

\[ I_e = k_a I_a + I_i \left[ k_d (N \cdot L)_+ + k_s (V \cdot R)^{n_s} \right] \]

\[ \text{L, N, V unit vectors} \]
\[ I_e = \text{outgoing radiance} \]
\[ I_i = \text{incoming radiance} \]
\[ I_a = \text{ambient light} \]
\[ k_a = \text{ambient light reflectance factor} \]
\[ (x)_+ = \max(x, 0) \]
Measuring the BRDF [5]

- Gonioreflectometer
  - Device for capturing the BRDF by moving a camera + light source
  - Need careful control of illumination, environment

E-mail: hogijung@hanyang.ac.kr
http://web.yonsei.ac.kr/hgjung
Measuring the BRDF [5]

- MERL (Matusik et al.): 100 isotropic, 4 nonisotropic, dense

  ![Measurement Image]

  - 20-80 million reflectance measurements per material
  - Each tabulated BRDF entails
    90x90x180x3=4,374,000 measurement bins

- CURET (Columbia–Utrect): 60 samples, more sparsely sampled, but also bidirectional texture functions (BTF)
Image Intensity and 3D Geometry [2]

- Shading as a cue for shape reconstruction
- What is the relation between intensity and shape?
  - Reflectance Map
For a point \( p \) on the surface, the image irradiance \( E(x,y) \) under a distant source is a function of

1. The BRDF at \( p \)
2. The surface normal at \( p \)
3. The direction of the light source
Reflectance Map [6]

Now if BRDF and light source direction/strength are known, then for an imag point \((x,y)\)

1. image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have \(E(p,q)\).
Surface Normal [2]

Equation of plane

\[ Ax + By + Cz + D = 0 \]

or

\[ \frac{A}{C} x + \frac{B}{C} y + z + \frac{D}{C} = 0 \]

Let

\[ -\frac{\partial z}{\partial x} = \frac{A}{C} = p \quad \text{and} \quad -\frac{\partial z}{\partial y} = \frac{B}{C} = q \]

Surface normal

\[ \mathbf{N} = \left( \frac{A}{C}, \frac{B}{C}, 1 \right) = (p, q, 1) \]
Gradient Space \[2\]

Normal vector

\[
\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}
\]

Source vector

\[
\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_S, q_S, 1)}{\sqrt{p_S^2 + q_S^2 + 1}}
\]

\[
\cos \theta_i = \mathbf{n} \cdot \mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2 + q^2 + 1} \sqrt{p_S^2 + q_S^2 + 1}}
\]

\(z = 1\) plane is called the Gradient Space (\(pq\) plane)

- Every point on it corresponds to a particular surface orientation
Reflectance Map [2]

- Relates image irradiance $I(x,y)$ to surface orientation $(p,q)$ for given source direction and surface reflectance
- Lambertian case:
  
  $k$: source brightness
  
  $\rho$: surface albedo (reflectance)
  
  $c$: constant (optical system)

  Image irradiance:
  
  $I = \frac{\rho}{\pi} kc \cos \theta_i = \frac{\rho}{\pi} kc \mathbf{n} \cdot \mathbf{s}$

  Let $\frac{\rho}{\pi} kc = 1$ then $I = \cos \theta_i = \mathbf{n} \cdot \mathbf{s}$
Reflectance Map [2]

- Lambertian case

\[ I = \cos \theta_i = \mathbf{n} \cdot \mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2 + q^2 + 1} \sqrt{p_S^2 + q_S^2 + 1}} = R(p, q) \]

Iso-brightness contour

Reflectance Map (Lambertian)

cone of constant \( \theta_i \)
Reflectance Map [2]

- Lambertian case

Note: $R(p, q)$ is maximum when $(p, q) = (p_s, q_s)$
Reflectance Map [2]

- Glossy surfaces (Torrance-Sparrow reflectance model)

\[ I = \frac{P_d}{\pi} k c \cos \theta_i + \frac{P_s k c}{\cos \theta_r} p(\beta)G = R(p, q) \]

- Diffuse term
- Specular term

Diagram:
- Diffuse peak
- Specular peak
- \( R(p, q) = 0.5 \)

E-mail: hogijung@hanyang.ac.kr
http://web.yonsei.ac.kr/hgjung
Shape from a Single Image? [2]

- Given a single image of an object with known surface reflectance taken under a known light source, can we recover the shape of the object?
- Given \( R(p, q) \) (\( (p, q) \) and surface reflectance) can we determine \( (p, q) \) uniquely for each image point?

Solution: Take more images
Photometric stereo
Shape from a Single Image? [3]

- Take more images ➔ Photometric stereo
- Add more constraints ➔ Shape–from–shading
Photometric Stereo [2]
Photometric Stereo [2]

Lambertian case:

\[ I = \frac{\rho}{\pi} k c \cos \theta_i = \rho \mathbf{n} \cdot \mathbf{s} \quad \left( \frac{k c}{\pi} = 1 \right) \]

Image irradiance:

\[ I_1 = \rho \mathbf{n} \cdot \mathbf{s}_1 \]
\[ I_2 = \rho \mathbf{n} \cdot \mathbf{s}_2 \]
\[ I_3 = \rho \mathbf{n} \cdot \mathbf{s}_3 \]

- We can write this in matrix form:

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix} = \rho
\begin{bmatrix}
\mathbf{s}_1^T \\
\mathbf{s}_2^T \\
\mathbf{s}_3^T
\end{bmatrix}
\mathbf{n}
\]
Solving the Equations [2]

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_2
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
s_1^T \\
\vdots \\
s_3^T
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
\rho \\
n
\end{bmatrix}
\begin{bmatrix}
S \\
\tilde{n}
\end{bmatrix}
\]

\[
\tilde{n} = S^{-1}I
\]

\[
\rho = \frac{\tilde{n}}{\tilde{n}}
\]

\[
n = \frac{\tilde{n}}{\rho}
\]
More than Three Light Sources [2]

- Get better results by using more lights

\[
\begin{bmatrix}
I_1 \\
\vdots \\
I_N
\end{bmatrix} = \begin{bmatrix}
S^T_1 \\
\vdots \\
S^T_N
\end{bmatrix} \rho \mathbf{n}
\]

- Least squares solution:

\[
\mathbf{I} = \mathbf{S} \tilde{\mathbf{n}} \quad \text{with} \quad N \times 1 = (N \times 3)(3 \times 1)
\]

\[
\mathbf{S}^T \mathbf{I} = \mathbf{S}^T \mathbf{S} \tilde{\mathbf{n}}
\]

\[
\tilde{\mathbf{n}} = (\mathbf{S}^T \mathbf{S})^{-1} \mathbf{S}^T \mathbf{I}
\]

- Solve for \(\rho, \mathbf{n}\) as before

Moore-Penrose pseudo inverse
Color Images [2]

- The case of RGB images
  - get three sets of equations, one per color channel:
    \[ I_R = \rho_R S_n \]
    \[ I_G = \rho_G S_n \]
    \[ I_B = \rho_B S_n \]

  - Simple solution: first solve for \( n \) using one channel
  - Then substitute known \( n \) into above equations to get
    \( (\rho_R, \rho_G, \rho_B) \)

  - Or combine three channels and solve for \( n \)
    \[ I = \sqrt{I_R^2 + I_G^2 + I_B^2} = \rho S_n \]
**Light Source Direction [2]**

- For a perfect mirror, light is reflected about \( \mathbf{N} \)

\[
R_e = \begin{cases} 
R_i & \text{if } \mathbf{v} = \mathbf{r} \\
0 & \text{otherwise}
\end{cases}
\]

- We see a highlight when \( \mathbf{v} = \mathbf{r} \)
- Then \( \mathbf{s} \) is given as follows:

\[
\mathbf{s} = 2(\mathbf{n} \cdot \mathbf{r})\mathbf{n} - \mathbf{r}
\]

\[
s = r - 2(r - (rn)n)
\]

\[
= r - 2r + 2(rn)n
\]

\[
= 2(rn)n - r
\]
Normal Field to Surface [6]

Many methods: Simplest approach

1. From normal field \( \mathbf{n} = (n_x, n_y, n_z) \), \( p = n_x/n_z \), \( q = n_y/n_z \)
2. Integrate \( p = df/dx \) along a row \((x,0)\) to get \( f(x,0) \)
3. Then integrate \( q = df/dy \) along each column starting with value of the first row
Normal Field to Surface [6]

Normal Field

Plastic Baby Doll: Normal Field

E-mail: hogijung@hanyang.ac.kr
http://web.yonsei.ac.kr/hgjung
Results [2]

1. Estimate light source directions
2. Compute surface normals
3. Compute albedo values
4. Estimate depth from surface normals
5. Relight the object (with original texture and uniform albedo)
Shape from a Single Image? [3]

- Take more images ➔ Photometric stereo
- Add more constraints ➔ Shape-from-shading
Human Perception [4]

• Our brain often perceives shape from shading.

• Mostly, it makes many assumptions to do so.

• For example:
  
  Light is coming from above (sun).

  Biased by occluding contours.
Stereographic Projection [4]

$$(p,q)$$-space (gradient space)

$$(f,g)$$-space

Problem

$$(p,q)$$ can be infinite when $\theta = 90^\circ$

Redefine reflectance map as $R(f,g)$
The \( \mathbf{n} \) values on the occluding boundary can be used as the boundary condition for shape-from-shading.
Image Irradiance Constraint [4]

- Image irradiance should match the reflectance map

Minimize

\[ e_i = \iint_{\text{image}} (I(x, y) - R(f, g))^2 \, dx \, dy \]

(minimize errors in image irradiance in the image)
Smoothness Constraint [4]

- Used to constrain shape-from-shading
- Relates orientations \((f, g)\) of neighboring surface points

Minimize

\[
es = \iiint (f_x^2 + f_y^2) + (g_x^2 + g_y^2) \, dx \, dy
\]

\((f, g)\): surface orientation under stereographic projection

\[
f_x = \frac{\partial f}{\partial x}, \quad f_y = \frac{\partial f}{\partial y}, \quad g_x = \frac{\partial g}{\partial x}, \quad g_y = \frac{\partial g}{\partial y}
\]

(penalize rapid changes in surface orientation \(f\) and \(g\) over the image)
Shape from Shading [4]

- Find surface orientations \((f, g)\) at all image points that minimize

\[
e = e_s + \lambda e_i
\]

weight

smoothness constraint

image irradiance error

Minimize

\[
e = \int \int_{\text{image}} \left( f_x^2 + f_y^2 \right) + \left( g_x^2 + g_y^2 \right) + \lambda \left( I(x, y) - R(f, g) \right)^2 \, dx \, dy
\]
Result [4]

by Ikeuchi and Horn
• In an environment where red, green, and blue light is emitted from different directions, a Lambertian surface will reflect each of those colors simultaneously without any mixing of the frequencies.
• The quantities of red, green and blue light reflected are a linear function of the surface normal direction.
For simplicity, we first focus on the case of a single distant light source with direction \( \mathbf{l} \) illuminating a Lambertian surface point \( P \) with surface orientation direction \( \mathbf{n} \).

\[
r_{i} = \mathbf{l} \cdot \mathbf{n} \int S(\lambda) \rho(\lambda) \nu_{i}(\lambda) d\lambda, \quad (1)
\]

- The intensity measured at \( i \)-th sensor
- The spectral sensitivity of the \( i \)-th sensor
- The Spectral reflectance function at that point
- The energy distribution of the light source
In matrix form

\[ \mathbf{r} = \mathbf{M} \mathbf{n}, \quad (2) \]

where the \((i, j)\)th element of \(\mathbf{M}\) is

\[ m_{ij} = l_j \int S(\lambda) \rho(\lambda) \nu_i(\lambda) d\lambda. \quad (3) \]

**RGB → normal mapping**: Equation (2) establishes a 1–1 mapping between an RGB pixel measurement from a color camera and the surface orientation at the point projecting to that pixel. Our strategy is to use the inverse of this mapping to convert a video of a deformable surface into a sequence of normal maps.

By estimating and then inverting the linear mapping \(\mathbf{M}\) linking RGB values to surface normals, we can convert a video sequence captured under colored light into a **video of normal-maps**.

We then integrate each normal map independently to obtain a depth map in every frame by imposing that the **occluding contour is always at zero depth**. At the end of the integration process, we obtain a **video of depth-maps**.
With Colored Lights [10]

Tracking the surface

Our approach is to use the first depth-map of the sequence as a 3D template which will be deformed to match all subsequent depth-maps.

The deformations of the template will be guided by the following two competing constraints:
- the deformations must be compatible with the frame to frame 2D optical flow of the original video sequence,
- the deformations must be locally as rigid as possible.
References