NOTES AND CORRESPONDENCE

On the Slow Mode of a Simple Air-sea Coupled Model: The Sensitivity to the Zonal Phase Difference between the SST and the Atmospheric Heating

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Abstract

An eigen analysis of the simple air-sea coupled model is carried out to understand the coupled mechanism with the slow time scale (so called 'slow mode') related to the zonal phase difference between the SST and atmospheric heating anomalies. Both frequency and instability of the slow mode are very sensitive to the zonal phase difference between the SST and atmospheric heating anomalies. When the positive atmospheric heating is located between one-quarter wavelength west and one-quarter wavelength east of the positive SST anomaly, the slow mode becomes the unstable eastward-propagating mode. The further eastward shifting of the atmospheric heating results in damping of the eastward-propagating mode resembling the forced Kelvin mode. On the other hand, the slow mode becomes the unstable westward-propagating mode when atmospheric heating lies farther west than one-quarter wavelength west from the SST anomaly. This unstable westward-propagating mode resembles the forced Rossby mode. SST changes in the eastward- and westward-propagating slow modes are mainly induced by the changes of the oceanic height and the zonal advection by the current, respectively. The slow mode in the fast-wave limit and the non-rotating system is also discussed.

1. Introduction

A great deal of research about the El Niño-Southern Oscillation (ENSO) has been done during the last decades. Moreover, the basic mechanisms of ENSO (e.g., Zebiak and Cane 1987; Battisti and Hirst 1989; Xie et al. 1989; Wakata and Sarachik 1991; Jin and Neelin 1993; Mantua and Battisti 1995; Jin 1997; Jin and An 1999) and their successful prediction (Ji et al. 1996; Kirtman and Zebiak 1997; Chen et al. 1998) have been documented. However, there are still uncertainties as to the origin of the irregularity, the phase and frequency locking to the annual cycle, and the interaction with other climate variabilities such as the monsoon and interdecadal variabilities. Among those, the basic mechanism that determines ENSO frequency is not fully solved yet.

Most studies to demonstrate physical mechanisms that determine the ENSO period have been performed by using the simplest possible models or the intermediate coupled models (e.g., Hirst 1986; Zebiak and Cane 1987; Battisti and Hirst 1989; Xie et al. 1989; Wakata and Sarachik 1991; Jin and Neelin 1993; Jin 1997; Kang and An 1998; An et al. 1999). However, the atmospheric models used in those studies are too simple to describe the realistic atmosphere response to SST anomalies. For instance, the atmospheric heating is assumed being proportional to the local SST anomaly. But in reality, the maximum region of SST anomalies is not matched to that of anomalous atmospheric heating. Even the tropical atmospheric wind anomalies may respond differently to a similar SST anomaly according to the season (Wang et al. personal communication), because atmospheric heating depends on not only the local SST anomaly but also on both atmospheric and oceanic basic states. Besides it is
well known that the SST and atmospheric heating are in a nonlinear relationship (Hirst 1986). In this sense, it is not easy to accurately describe the atmosphere response to the SST anomaly. Even Zebiak and Cane (1987)'s model, in which the nonlinear coupling process between the ocean and atmosphere is adopted, showed the prominent discrepancy with the observation in the zonal pattern (Perigaud and Dewitte 1996).

In the present study, we investigate the sensitivities of the coupled mode with the ENSO time scale, the so-called 'slow mode' (Wang and Weisberg 1994), to the zonal phase difference between the SST and the atmospheric heating and its associated dynamics. The slow mode has been documented as a good prototype for slowly propagating modes in a number of intermediate models and GCMs (e.g., Anderson and McCreary 1985; Lau et al. 1992).

2. Model

In this study, in order to describe the ENSO-like interannual variation, a linear, equatorial $\beta$-plane, reduced gravity model is used. Here, the zonal phase difference ($\theta$) between SST and the atmospheric heating is prescribed. The governing equations for the atmosphere and ocean, which were in advance non-dimensionalized with respect to a length scale of $(C_0/\beta)^{1/2}$ and a time scale of $(C_0, \beta)^{-1/2}$, can be written as follows:

**ATMOSPHERE:**

\[
AU - yV + \frac{\partial \Theta}{\partial x} = 0 \tag{1a}
\]

\[
AV + yU + \frac{\partial \Theta}{\partial y} = 0 \tag{1b}
\]

\[
B\Theta + C^2 \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) = -Q_a \tag{1c}
\]

Where $U$ and $V$ are the anomalous zonal and meridional winds in the lower atmosphere, respectively; $\Theta$ the pressure perturbation; $A$ and $B$ the Rayleigh friction and Newtonian cooling coefficients, respectively; $C$ the ratio between the atmospheric and oceanic gravity wave speed, defined as $C_a/C_0$; $Q_a$ the atmospheric heating parameterized as $K_Q e^{-\theta T}$.

**OCEAN:**

\[
\frac{\partial u}{\partial t} - yv + \frac{\partial h}{\partial x} = \tau - au \tag{2a}
\]

\[
\frac{\partial v}{\partial t} + yu + \frac{\partial h}{\partial y} = -av \tag{2b}
\]

\[
\frac{\partial h}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -ah \tag{2c}
\]

Where $u$ and $v$ are the zonal and meridional current anomalies, respectively; $h$ the upper-layer thickness perturbation; $\tau$ the zonal wind stress anomaly parameterized as $\gamma U$; $a$ and $b$ the coefficients of Rayleigh friction and Newtonian cooling, respectively; $\lambda$ and $\sigma$ the reversed zonal gradient of mean SST and the entrainment thermal forcing coefficient, respectively. The coefficients used in this model are summarized in Table 1.

Eigenvector analyses are performed for the coupled model of Eqs. (1) and (2). Solutions are sought of the sinusoidal form in the longitude and time $e^{i(kx-\omega t)}$, where $k$ is the zonal wave number and $\omega$ is an eigenvalue. Both north and south meridional boundaries are $53^\circ S$ and $53^\circ N$, respectively. Details of the numerical method are in Hirst (1986) and Wakata and Sarachik (1992).

3. Results

The present coupled model has three major eigenmodes: the forced Kelvin mode, the forced Rossby mode and the slow mode ('U-mode' in Hirst 1986). Among them, the slow mode has the phase speed of the interannual time scale, and other two forced modes have phase speeds that are similar to those of the corresponding free modes. The slow mode might be produced by a combination of two fast propagating modes (Hirst 1986), i.e. forced Kelvin and Rossby modes. The present study focuses on these three modes.

Figure 1 shows the changes of the growth rate and frequency of three eigenmodes (forced Kelvin and Rossby modes and slow mode) when $\theta$ is changed. Here, the positive and negative $\theta$ indicate the east and west shift of the atmospheric heating with respect to SST anomaly, respectively. As shown in Fig. 1, both growth rate and frequency are very sensitive to $\theta$, particularly of the slow mode. The periods of the forced Kelvin and Rossby modes, which always propagate eastward and westward, respectively, vary within 2–4 months and 7–11 months, respectively. On the other hand, both eastward- and westward-propagating slow modes have a longer period more than 8 months. In other words, the periods of both forced Kelvin and Rossby modes are similar to the basin crossing time of the equatorial free Kelvin and Rossby waves, respectively; and only the slow mode has the slowly propagating speed of the interannual time scale with the strong growth rate, which occurs when the atmospheric heating is centered around one-quarter wavelength west of the SST anomaly ($-60^\circ < \theta < -10^\circ$). In some sense, these results are consistent with the observation showing that the anomalous atmospheric heating was displaced typically between $10^\circ$ and $40^\circ$ of
longitude west from anomalous SST (Hirst 1986), because for the corresponding range of a the coupled model has the unstable mode with the interannual time scale. Note that the changes in direction of the moving SST anomaly during ENSO may be related to the change of a caused by the different background states or by the nonlinear interaction between the SST and atmospheric heating, because the slow mode can be either the westward- or eastward-propagating mode within the a-range having the unstable interannual time scale mode.

As the atmospheric heating is located successively farther to the west, the slow mode becomes the westward propagation mode and eventually mode its frequency is close to that of the forced Rossby at near a = -120°. In contrast, as the atmospheric heating successively shifts to the east, the frequency of the slow mode becomes similar to that of the forced Kelvin mode at near a = 60°. The slow mode goes into the damped regime by the eastward shifting of the atmospheric heating and the further shift to the east results in the destabilization of both forced Kelvin and Rossby modes. Note that the sensitivities of the eigenmode to the change of a in the fast wave limit are very similar to those of the slow mode in the present model (see Fig. 1). The fast wave limit indicates that the ocean dynamic fields very quickly adjust to a given forcing and it is done by eliminating the time dependent terms in the ocean dynamics equations. The eigenmode in the fast wave limit has been called the ‘slow SST mode’ (Jin and Neelin 1993). Although the slow mode in the present study and the slow SST mode are not exactly same, we may say that the thermodynamic adjustment process is the primary cause to drive the slow mode in the present model.

When -40° < a < 150°, the slow mode propagates to the east, and when a < -40°, the slow mode propagates to the west (see Fig. 1). In order to address the dynamics related to such changes in the propagating direction, the spatial patterns of the zonal wind, SST, the oceanic height, and current

Table 1. Basic parameters and their values used in the present model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol and values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meridional gradient of Coriolis parameter</td>
<td>(\beta = 2.28 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1})</td>
</tr>
<tr>
<td>Atmospheric Rayleigh friction</td>
<td>(A = (2 \text{ day})^{-1})</td>
</tr>
<tr>
<td>Atmospheric Newtonian cooling rate</td>
<td>(B = (2 \text{ day})^{-1})</td>
</tr>
<tr>
<td>Oceanic Rayleigh friction</td>
<td>(a = (2.5 \text{ year})^{-1})</td>
</tr>
<tr>
<td>Oceanic Newtonian cooling rate</td>
<td>(\theta = (125 \text{ day})^{-1})</td>
</tr>
<tr>
<td>Atmospheric gravity wave speed</td>
<td>(C_a = 0.0 \text{ m}^{-1})</td>
</tr>
<tr>
<td>Oceanic gravity wave speed</td>
<td>(C_a = 2.9 \text{ m}^{-1})</td>
</tr>
<tr>
<td>Wind stress coupling coefficient</td>
<td>(\gamma = 1.6 \times 10^{-7} \text{ s}^{-1})</td>
</tr>
<tr>
<td>Background zonal SST gradient</td>
<td>(\lambda = 5.0 \times 10^{-7} \text{ C m}^{-1})</td>
</tr>
<tr>
<td>Entrainment thermal forcing coefficient</td>
<td>(\sigma = 5.0 \times 10^{-9} \text{ K m}^{-1} \text{ s}^{-1})</td>
</tr>
<tr>
<td>SST-evaporation coefficient</td>
<td>(K_0 = 7.0 \times 10^{-3} \text{ m}^3 \text{ s}^{-3} \text{ K}^{-1})</td>
</tr>
<tr>
<td>Mean upper layer depth</td>
<td>(H = 150 \text{ m})</td>
</tr>
<tr>
<td>Reduced gravity</td>
<td>(\mathcal{G}' = 0.056 \text{ m}^{-1})</td>
</tr>
</tbody>
</table>

Fig. 1. Growth rate (dashed line) and frequency (solid line) as functions of the zonal phase difference between the atmospheric heating and SST anomalies (a). Colors of the line refer to the forced Kelvin mode (red), the forced Rossby mode (green) and the slow mode (black). Both growth and frequency are in nondimensional units, \(10^{-2} = (140 \text{ days})^{-1}\). In this calculation, the nondimensional wave number of 0.11 is used.

Fig. 2. As in Fig. 1 but for the fast wave limit.
obtained from the eigenvectors when $\theta = -60^\circ$, $0^\circ$, and $+60^\circ$, are shown in Fig. 3. The oceanic height and the current of the westward-propagating slow mode ($\theta = -60^\circ$ case, the lower panel of Fig. 3a) resemble those of the free Rossby wave. And those of the eastward propagating slow mode ($\theta = +60^\circ$ case, the lower panel of Fig. 3c) resemble those of the free Kelvin wave. The eigenvector for $\theta = 0^\circ$ (Fig. 3b) shows an intermediate spatial structure. The changes of the eigenvectors due to the changes of $\theta$ are consistent with those in eigenvalues as shown in Fig. 1.

As shown in Fig. 3a, the oceanic height of the westward propagating slow mode appears slightly off the equator, and the zonal current along the equator is relatively strong, due to not only the zonal wind stress but also the geostrophic current induced by the meridional gradient of the oceanic height. As a result, the SST tendency due to the change of the oceanic height (so-called 'thermocline feedback') is weaker than that due to the thermal advection by the anomalous zonal current (so-called 'zonal advection feedback'). Thus, the positive SST tendency due to both thermocline and zonal advection feedbacks is located slightly to the east of the positive SST anomaly and the maximum of the oceanic height appears along the equator. In this case (see Fig. 3c), the oceanic height and zonal current anomalies are almost in phase. Consequently, the slow mode propagates to the east.

Although the stability for the different $\theta$ is directly calculated in Fig. 1 such that the eigenmodes for $\theta = -60^\circ$ and $\theta = 0^\circ$ are unstable and that for $\theta = +60^\circ$ is stable, the stability for each eigenmodes may also be realized from the spatial distributions of the wind stress and the zonal current. As discussed by Yamagata (1985), a necessary condition of the tropical atmosphere-ocean coupled instability is that the current and the wind must be in the same direction. As shown in Fig. 3, the wind and current of the unstable modes ($\theta = -60^\circ$ and $\theta = 0^\circ$) are almost in the same direction, whereas those of the stable mode ($\theta = +60^\circ$) are almost in the opposite direction. These results are consistent with Yamagata's.

Figure 4 shows the growth rate and frequency of the slow mode on the $k$-$\theta$ plane. The $\theta$ creating the maximum growth rate (hereafter, $\theta_M$) is about $60^\circ$ at $k = 0.06$. In the long wave regime, $\theta_M$ becomes relatively larger and in the short wave regime, $\theta_M$
converges to about $-30^\circ$. In this sense, the $\theta_M$ is a function of the zonal wave number (Xie et al. 1999).

The frequency of the interannual time scale appears at $\theta = -30^\circ$ for the shorter wave ($k > 0.1$). As the zonal scale becomes larger, $\theta$ of the slow mode having the frequency of the interannual time scale also increases.

By removing the rotational components and the time dependent terms of the ocean dynamics from Eqs. (1) and (2), the dependency of the slow mode on $\theta$ under the non-rotating fast-wave limit can be addressed. The corresponding dispersion relationship is following:

$$\omega_R = \frac{k\gamma KQ (k\sigma \sin \theta - a\lambda \cos \theta)}{(a^2 + k^2)(AB + k^2C^2)}$$  \hspace{1cm} (3a)$$

$$\omega_I = \frac{k\gamma KQ (k\sigma \cos \theta + a\lambda \sin \theta)}{(a^2 + k^2)(AB + k^2C^2)} - b$$ \hspace{1cm} (3b)

Here, $\omega_R$ and $\omega_I$ are the frequency and growth rate, respectively. In this system, $\theta_M = \tan^{-1}[a\lambda/(k\sigma)]$ from the necessary condition for the maximum growth rate that the partial derivative of $\omega_I$ with respect to $\theta$ is zero. This equation allows that $\theta_M$ approaches $(2N - 1)\pi/2$ ($N$ is integer) for the large scale wave and $\theta_M$ goes to $N\pi$ for the small scale wave. It also shows that the increase of the thermocline feedback effect $\sigma$ results in $\theta_M = N\pi$ and the increase of the zonal advection feedback effect $\lambda$ results in $\theta_M = (2N - 1)\pi/2$. From Eq. (3a), one can also calculate the value of $\theta$ for $\omega_R = 0$ such that $\theta_0 = \tan^{-1}[a\lambda/(k\sigma)]$. This equation shows that for the large-scale wave, $\theta_0$ approaches $(2N - 1)\pi/2$ and for the small-scale wave, $\theta_0$ approaches $N\pi$. Note that the stationary wave has the maximum growth rate, since the definitions of $\theta_0$ and $\theta_M$ are exactly same. On the other hand, the present model as shown in Fig. 4 shows that the growth rate tends to be maximized at about $\theta = -30^\circ$ as the wave scale becomes smaller and at about $\theta = +60^\circ$ as the wave scale becomes larger. The stationary wave (i.e. zero frequency line) in Fig. 4b appears when about $\theta = -30^\circ$ for the small-scale wave, and the corresponding $\theta$ goes up to about $+90^\circ$ as the wave scale increases. As discussed above, $\theta_M$ and $\theta_0$ in such a highly simplified system shows a different behavior to those of the present model. The major difference between the two systems might be due to the non-rotating effect rather than the fast-wave limit. From Eq. (3) and Fig. 4, one can estimate the difference between the $\theta_M$ and $\theta_0$ of the non-rotating system and those of the present model, which is about $30^\circ$. Thus, by shifting $\theta$ in Eq. (3) by $30^\circ$, the $\theta_M$ and $\theta_0$ of the non-rotating system are close to those of the rotating system in terms of the sensitivity to the wave scale. In other words, the removing of the rotation effect results in the $30^\circ$-phase difference.

As investigated in this study, all major characteristics of the eigenmodes such as instability, frequency, and propagating direction are highly dependent on $\theta$. Thus, the accurate modeling of the atmospheric heating parameterization and SST dynamics is very important in view of determining the frequency and growth rate of the coupled model.

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**References**


簡単な大気海洋結合モデルのスローモードについて：
海面水温と大気加熱の間の東西方向の位相差に対する感度

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長い時間スケールを持つ結合モード（いわゆる“スローモード”）のメカニズムを海面水温と大気加熱の間の東西方向の位相差に関連して理解するために、大気海洋結合モデルの固有モード解析をおこなった。スローモードの振動数および安定性の双方とも海面水温と大気加熱の間の東西方向の位相差に大変敏感である。正の大気加熱が正の海面水温偏差に対し西1/4波長から東1/4波長の間に位置するとき、スロー モードは東へ伝播する不安定モードとなる。大気加熱がこれより東方へずれると、強制ケルビン波に似た東進波は減衰モードとなる。一方、加熱が海面水温より1/4波長以上西にずれるとスローモードは西へ伝播する不安定モードとなる。この西進波は強制ロスピーモードに似ている。東進および西進スローモードの海面水温変化をもたらす主要因は、それぞれ圏層深度および海流による東西移流の変化である。海洋の赤道波の位相速度を無限大とするいわゆる“fast-vave リミット”および非回転系の近似のもとでのスロー モードについても言及される。