Spatial information of high-frequency brightness temperatures for passive microwave rainfall retrievals

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The effect of rainfall inhomogeneity within the sensor field of view (FOV) affects significantly the accuracy of rainfall retrievals causing the so-called beam-filling error. Observational analyses of Tropical Rainfall Measuring Mission (TRMM) Microwave Imager (TMI) and Precipitation Radar (PR) data suggest that the beam-filling error can be classified in terms of the mean rain rate and the rainfall inhomogeneity parameter or coefficient of variation (CV_R, standard deviation divided by mean). The dependence of the beam-filling error on the rain rate and CV_R has been confirmed quantitatively using a single channel at 19.4 GHz. It is also found significantly different beam-filling errors for the two different regions, the East and West Pacific, where the spatial and vertical distributions of rainfalls are different. It is also observed that the vertical distribution of rainfall is related to the spatial variability of rainfall (CV_R) and similarly to the spatial variability of TMI 85.5 GHz brightness temperature (CV_Tb). Based on these findings, this study exploits the CV_Tb to correct the beam-filling error in a direct inversion from a rainfall (R) and brightness temperature (T_b) curve at a single frequency, and to reduce the retrieval error in the context of a Bayesian-type inversion method for multi-frequency rainfall retrievals. Both the experiments suggest that the spatial variability of the high-frequency radiometer data appears to contain useful information for retrievals.

1. Introduction

Microwave radiometry from low Earth orbiting satellites has been used extensively to provide global cloud and rainfall information. Despite its discrete sequence of visits over a given location, unlike infrared and visible sensors at a geostationary orbit, the overwhelming advantage of microwave radiometry in cloud transmission properties for measuring the blackbody emission from cloud columns has been successfully demonstrated under all-weather conditions in remote sensing of geophysical parameters.

In the estimation of rainfall from microwave radiometers we can broadly classify two types of retrievals based on the radiometric signatures on the scattering or emission property of hydrometeors. The scattering-based algorithms are usually based on statistical approaches correlating the depression of radiance with the amount of scattering ice particles (Spencer et al. 1989, Grody 1991, Ferraro and Marks 1995). It is known that a weak link between frozen hydrometeors and liquid contents near the
surface at the scattering frequencies produces uncertainties of estimations. However, it seems that the scattering signature is the only way to measure rainfalls over land where the emission signals from raindrops and background are not quite discernible due to the high and variable emissivity of land. A rainfall retrieval model based on the thermal emission from raindrops was first developed by Wilheit et al. (1977). In the early stage of the development, rainfalls are inverted directly from the observed radiances using a simple one-dimensional cloud model. The need for multidimensional cloud models in physically based retrieval algorithms was continuously emphasized by a number of investigators (e.g. Mugnai and Smith 1988, Smith and Mugnai 1988, Adler et al. 1991, Smith et al. 1992, Mugnai et al. 1993, Kummerow et al. 1996) and thus the inversion with the multidimensional cloud model encourages using multichannel radiometric signatures simultaneously.

Regardless of the level of sophistication in the algorithms, however, three types of retrieval errors commonly exist: random, sampling and algorithm errors. Random errors refer to random noise of the sensor. Sampling error arises because only part of the rain field is sampled. The irregular sampling characteristics cause errors in estimations of spatial and temporal averages. The sampling error may be also associated with significant spatial variability and vertical heterogeneity of rainfall over low-resolution satellite footprints. Algorithm errors are associated with uncertainties of the algorithm physics and retrieval methods, arising from various sources such as the details of forward computation in conjunction with assumed microphysical properties of hydrometeors and three-dimensional radiative effect of precipitating system, and so on.

Among the algorithm errors, the beam-filling error, which arises because of a coupling between the inhomogeneity of the rain field within the sensor field of view (FOV) and the non-linear response of the observed microwave brightness temperature ($T_b$) to rain rate ($R$), is considered a major uncertainty. The importance of the beam-filling error has been stressed by a number of investigators (Chiu et al. 1990, Short and North 1990, Ha and North 1995). From theoretical consideration and empirical studies, Chiu et al. (1990) examined radar data collected by the Global Atmospheric Research Program (GARP) Atlantic Tropical Experiment (GATE) using a $T_b - R$ relation derived from radiative transfer calculations based on an atmospheric model of Wilheit et al. (1977). Chiu et al. (1990) proposed a first-order approximation of the beam-filling error. The beam-filling error is dependent on the rain rate variance within the FOV coupled to algorithm physics and sensor response of the form

$$[R] - R_E = -\frac{1}{2} \left\{ (R - \langle R \rangle)^2 \times \frac{T_b''([R])}{T_b'([R])} \right\},$$

where $R_E$ is the estimated rain rate, $[R]$ is the true rain rate with [] representing FOV average and $T_b''$ and $T_b'$ are, respectively, the curvature and slope of the $T_b - R$ relation. Since $T_b' > 0$ and $T_b'' < 0$, and the variance of rain rate $(R - \langle R \rangle)^2 > 0$, the right-hand side (RHS) is greater than 0. The estimated rain rate is therefore statistically less than the true rain rate (negative bias). The RHS can then be decoupled as a part that depends only on the algorithm physics and sensor response ($T_b''/2T_b'$) and a part that depends only on the rain field variability $(R - \langle R \rangle)^2$.

There are a number of rainfall products that are directly impacted by the beam-filling correction. The Wilheit et al. (1991) technique, which applies a beam-filling correction factor (BFC) obtained from the first-order formula and the GATE data, has
been used to retrieve oceanic rainfall that forms the basis of the Global Precipitation Climatology Project (GPCP) products (Huffman et al. 1997). Their technique has been applied to Tropical Rainfall Measuring Mission (TRMM) Microwave Imager (TMI) data to generate the standard TRMM product (3A11).

The beam-filling correction is not usually treated explicitly in instantaneous rainfall retrievals with multi-frequency such as the Goddard Profiling Algorithm (GPROF, Kummerow et al. 2001), which produces the TRMM 2A12 product. Instead, the GPROF uses a convective–stratiform (C–S) discrimination. The C–S discrimination is based on a combination of the texture information (horizontal gradient) based on the work of Hong et al. (1999) and Olson et al. (2001) and polarization at 85 GHz (Spencer et al. 1989). Wang (1995) discussed that convective rains are associated with a higher freezing height and a higher rain rate variance. This is consistent with the notion that a higher rain rate variance is associated with convective rain rather than stratiform rain, hence a higher beam-filling correction is required. Based on the argument, using the C–S separation may contribute to reduce the beam-filling error. The connectivity between spatial variability and vertical distribution of rainfall will be discussed in §3.

The data acquired by the TRMM satellite provide a unique opportunity to assess the beam-filling uncertainty associated with space-borne passive microwave rainfall estimation. The TRMM provides almost concomitant observations of microwave radiometer and space-borne radar. In this study, collocated observations of the microwave $T_b$ from the TMI and rainfall profile retrieved from the TRMM Precipitation Radar (PR) are examined for different climate regimes. This study addresses the observational evidences describing the problem of the rainfall inhomogeneity and introduces the spatial variability of $T_b$ at a high-frequency channel to correct the beam-filling error in rainfall estimations. It is also demonstrated that the spatial information can be easily incorporated into a Bayesian inversion scheme for instantaneous rainfall retrievals. An advantage of this observational approach is that radiometric responses to the uncertainties due to the rain field itself can be retrieved without intervention of additional uncertainties from the other sources such as those that may be introduced by radiative transfer calculations. In addition, the successful operation of the TRMM satellite, now in its 12th year, allows us to construct reliable information about the regional and temporal variation of the radiometric signatures to rainfall structures. The information, thus, can serve as the basis of a more comprehensive error model to improve passive microwave rainfall retrievals.

2. Data and inhomogeneity parameters

The TMI measures upwelling microwave radiances (brightness temperatures) emitted by the Earth and atmosphere at the five frequencies, 10.7, 19.4, 21.3, 37.0 and 85.5 GHz with horizontal and vertical polarizations (only vertical polarization at 21.3 GHz). The radiometer views the atmosphere at an oblique angle (49°) and corresponding incident angle of 52.8° at the surface of the Earth. The swath width of the radiometer is 758.5 km and each frequency has different FOVs, which are determined by the satellite altitude, antenna size and beam width. The effective FOV at 10.7 GHz is 63 km (down-track direction) by 37 km (cross-track direction), and the other frequencies’ (19.4, 21.3, 37.0 and 85.5 GHz) FOVs are $30 \times 18$ km, $23 \times 18$ km, $16 \times 9$ km and $7 \times 5$ km, respectively. The PR operates at 13.8 GHz and measures the return power with a vertical spacing of 250 m for normal samples at a nadir resolution
of ∼4 km and a swath width of 215 km. The details of the TRMM instruments can be found in Kummerow et al. (1998).

This study uses the collocated data sets of TMI brightness temperature ($T_b$, 1B11 in the TRMM product classification) and PR rainfall profile (2A25). The PR rainfall data are obtained from the relationships between measured radar reflectivity and rain rate (Iguchi et al. 2000). As typical radar rainfall measurements, the PR rainfall data are sensitive to drop size distribution (DSD) parameters. The cross-track scanning of the PR and the conical scanning of TMI introduce less than a minute and a half of time lag between the PR and TMI pixels. For a given FOV, the FOV-averaged rainfall $[R]$ are simply obtained by

$$[R] = \frac{1}{N} \sum_{i=1}^{N} R_i,$$

where $R$ indicates the PR near-surface rainfall at the resolution of ∼4 km and $N$ is the number of PR sub-grids within the FOV. The 10.7, 19.4 and 37.0 GHz resolutions of the TMI are mainly used. The analyses in this study are carried out over sections of the East Pacific (5° N–15° N, 150° W–120° W) and the West Pacific (130° E–160° E, 2° N–12° N) during the period of December 1999 to February 2000.

The radiometric signatures of low-frequency channels such as 10.7 and 19.4 GHz are mostly determined by emission properties of hydrometeors along the vertical path due to their great transmission capability through clouds. The upwelling brightness temperatures (or radiances) are, therefore, modulated by both the amounts of cloud hydrometeors and their vertical distributions. Furthermore, the combined effect of spatially inhomogeneous rainfall and coarse resolutions of low-frequency channels contributes to more variability on the emission signatures, resulting in a major ambiguity in rainfall retrievals. This section investigates how the radiometric signatures vary on the spatial variability of rainfall and discusses the spatial variability and vertical distribution of rainfall along with the spatial information from high-frequency $T_b$.

The spatial variability of a field for a given area may be described by the coefficient of variation (CV), which is defined by the ratio of standard deviation to mean. This study uses spatial information from PR surface rainfall and TMI brightness temperature at 85.5 GHz (horizontal polarization). The CV of rainfall within a FOV (CV$_R$) is obtained by

$$CV_R = \frac{\left( \frac{1}{N} \sum_{i=1}^{N} (R_i - [R])^2 \right)^{0.5}}{[R]},$$

and the CV of $T_b$ at 85.5 GHz within a FOV (CV$_{Tb}$) is also computed from

$$CV_{Tb} = \frac{\left( \frac{1}{N} \sum_{i=1}^{N} (T_{bi} - [T_b])^2 \right)^{0.5}}{[T_b]},$$

where square brackets denote FOV average as before.
The observed relationships between \( T_b \) and \( R \) at the three different frequencies as a function of CV\( R \) for two different climate regimes, the East and West Pacific, are presented in figure 1(a),(c),(e) and in figure 1(b),(d),(f), respectively. The concave downward curves grouped by CV\( R \) appear to be well separated and show faster saturation for larger rainfall inhomogeneity. Generally, the differences between the curves are significant with increasing rain intensity at the frequencies where the effect of emission is dominant (10.7 and 19.4 GHz). It also suggests that the high rain rates greater than about 10 mm hour\(^{-1}\) are usually accompanied by low (CV\( R < 1 \)) and medium (1 \( \leq \) CV\( R < 2 \)) rainfall inhomogeneity over the East Pacific. Meanwhile, the high rain rates are usually associated with the low rainfall inhomogeneity over the West Pacific. The spatial inhomogeneity of rainfall also affects greatly the \( T_b - R \) relations at 37.0 GHz until saturation reaches. As inferred from the curves on different rainfall inhomogeneity, a given temperature can be associated with different rainfalls. This non-uniqueness in \( T_b - R \) matching resulting from the rainfall inhomogeneity causes the beam-filling error in retrievals. One can expect that the beam-filling bias (underestimate) increases with increasing CV\( R \). The beam-filling error will be quantified based on a single-channel approach in §3.

It is examined whether the spatial inhomogeneity is associated with the vertical structures of rainfall. Figure 2 shows the vertical profiles of rain rate at the different CV\( R \)s and two FOVs (10.7 GHz-FOV and 19.4 GHz-FOV) over the East Pacific when the 2 km rain rate is about 7.5 mm hour\(^{-1}\) (figure 2(a) and (c)). It can be seen that the mean profiles are distinguished by the inhomogeneity parameter. The rain profiles associated with lower CV\( R \) tend to show the existence of the bright band, which is indicative of stratiform rain, whereas the profiles of high CV\( R \) tend to show no bright band, indicative of convective precipitation. The mean rainfall profiles with different rain rates reveal a similar pattern (not shown). This suggests the fact that the beam-filling bias is due to the coupling between the spatial inhomogeneity within FOVs and associated vertical structures.

It is also investigated whether the spatial information at the 85.5 GHz (CV\( T_b \)) can discriminate the vertical structures of rainfall. Figure 2(b) and (d) shows the average vertical PR rain profile classified according to the CV\( T_b \). The range of CV\( T_b \) (0–0.15) is much smaller than that of CV\( R \) (0–4), presumably due to the large mean \( T_b \) at 85.5 GHz. It can be seen that both indices are equally capable of separating the convective and stratiform profiles. This indicates that there is sufficient information in CV\( T_b \) for classifying convective and stratiform rain, which is responsible for a large fraction of the beam-filling error variability. The separation of rainfall vertical structures by CV\( T_b \), however, appears to be less prominent at low rain rates (less than about 2–3 mm hour\(^{-1}\)).

3. The beam-filling error

As discussed in §2, one can expect that the beam-filling error may vary as a function of the mean and CV\( R \) (equivalently, variance) of rainfall at a given FOV. This section quantifies the BFC for different rainfall inhomogeneities from a single channel (19.4 GHz) and suggests the use of the spatial information of high-frequency \( T_b \) for the beam-filling error correction.
Figure 1. Relationships between brightness temperature ($T_b$) and Precipitation Radar (PR) surface rain rate ($R$) at the three different channels (horizontally polarized) of Tropical Rainfall Measuring Mission (TRMM) Microwave Imager (TMI) for the different intervals of $CV_R$ over the East (a, c, e) and West (b, d, f) Pacific FOV, field of view.
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3.1 $T_b$ responses to homogeneous rainfalls

As shown in the first-order formula for the beam-filling error (equation (1)), the beam-filling error is first depending on the curvature of the $T_b$–$R$ relation, which is determined by radiative transfer computation given hydrometeor profiles and surface parameters from cloud models. Formulation of an appropriate $T_b$–$R$ relationship for a homogeneously raining FOV is therefore important. In this study, instead of using a relation obtained by applying only different freezing levels to the same cloud model, the $T_b$–$R$ curves for homogeneous rainfalls are computed using simulated rain profiles over two different climate regimes, the East and West Pacific. The $T_b$–$R$ relations
therefore will be determined by the regional characteristics, such as the columnar rain drops, freezing level and so on.

For the simulation of rain profiles, a methodology introduced by Shin and Kummerow (2003) is used. The simulation method was proven to show good physical consistency in the observed and computed $T_b$s at the emission channels. A brief description about the simulation method follows. The precipitating region is first identified by the PR near-surface rain in the TMI swath. If a rain signal is detected, the raining profile that best fits the PR reflectivity profile is selected from the cloud-resolving model (CRM) simulations. The reflectivity of the cloud-model profiles was obtained by computing single-particle backscattering properties based on Mie theory and assuming the same Marshall–Palmer (Marshall and Palmer 1948) DSDs that are used in the CRMs as follows:

$$Z = \frac{\lambda^4}{\pi^5 |K|^2} \int_D \sigma_b(D)N(D)\,dD,$$

(5)

where $\lambda$ is wavelength, $K = (m^2 - 1)/(m^2 + 1)$ is the dielectric factor of the scattering particle, $m$ is the complex index of refraction, $\sigma_b(D)$ is the backscattering cross section as a function of the particle’s effective diameter and $N(D)$ is the number density of particles.

The models used in this study are the Goddard Cumulus Ensemble (GCE) model and the University of Wisconsin Non-hydrostatic Modelling System (UW-NMS). A description of the GCE model can be found in Tao and Simpson (1993). The cloud microphysics includes five different hydrometeors (cloud water, rain, cloud ice, snow and graupel). The distributions of rain, snow and graupel are taken to be inverse exponentials of the form prescribed by Marshall and Palmer (1948), but with an intercept dictated by the physical parameterizations. The horizontal resolution of these models varies from 1 to 2 km, while the vertical coordinate is 500 m in the lower troposphere. The second model used is UW-NMS described by Tripoli (1992a). Aside from differences in the dynamical assumptions in the model, the UW-NMS considers four classes of ice – graupel, pristine crystals, snow crystals and aggregates – that need to be treated differently from the GCE profiles. A detailed description of these ice categories and their interaction may be found in Tripoli (1992b). Horizontal and vertical coordinates are similar to those of the GCE simulations.

The radiative transfer computations using the selected CRM profiles are performed using a plane parallel theory and Eddington approximation (Kummerow 1993). As demonstrated in Shin and Kummerow (2003), $T_b$s calculated for the raining scene turns out to be reasonably close to actual observations at the frequency. Figure 3(a) shows the computed $T_b$s at 19.4 GHz (horizontal polarization) as a function of rain rates for the East and West Pacific. The exponentially fitted curves are superimposed. It can be seen that the mean curve for the West Pacific has warmer $T_b$s below about 10 mm hour$^{-1}$, suggesting more liquids in the column. The warmer temperatures are also due to higher freezing height in the West Pacific. Figure 3(b) compares the mean observed $T_b$–$R$ curves for the two regions. The upper and lower error bars show the mean ±1 standard deviation for the same rain rate category. The observed $T_b$–$R$ curve shows a lower $T_b$ for the same rain rate, demonstrating the classical beam-filling problem in microwave remote sensing of rain. The difference of the relationship between the two regions is prominent with increasing rain intensity (above 6–7 mm hour$^{-1}$). Here one may note that the rain-layer thickness is a major discriminator below about 4–5 mm hour$^{-1}$, but uncertainty from inhomogeneous rainfall seems to have greater
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Figure 3. The relationships between brightness temperatures \(T_b\) and rain rates \(R\) for 19.4 GHz horizontally polarized channel. (a) \(T_b-R\) curves under the assumption of homogeneous rainfall within the 19.4 GHz-FOV. A plane parallel radiative transfer model is used to compute \(T_b\)'s with the cloud–resolving model (CRM) profiles selected to best match Precipitation Radar (PR)-observed reflectivity profiles. The curves fitted to the exponential function are also presented in solid (East Pacific) and dashed (West Pacific) lines. (b) \(T_b-R\) curves obtained from the collocated PR and Tropical Rainfall Measuring Mission (TRMM) Microwave Imager (TMI) data. The error bars indicate the mean ± 1 standard deviation. FOV, field of view.

contribution to the difference above these rain rates. The West Pacific is characterized by a higher freezing level than that of the East Pacific. This indicates that the West Pacific shows warmer \(T_b\)'s with the same rain intensity than the East Pacific and it appears to be applicable to rain rates from 0 to about 4–5 mm hour\(^{-1}\). However as rain intensity increases, the larger inhomogeneity factor for the West Pacific compared with that of the East Pacific turns out to produce more non-linearity in the relationship. It suggests that rainfall estimation based on emission property can have serious biases (consistent underestimates) over the regions with the considerable rain inhomogeneity in the FOV if it is not appropriately accounted for (Wilheit 1986).

3.2 BFC and \(CV_{T_b}\)

Figure 4 shows the mean BFC, the ratio of the true rain to the estimated rain, obtained at the rain rate intervals (1 mm hour\(^{-1}\)) and three different categories of \(CV_R\). The BFCs for all the data without sub-grouping by \(CV_R\) are also presented (indicated by the symbol '*'). The PR rain rates at the FOV of 19.4 GHz are assumed as truth and estimated rain rates are obtained by inverting observed \(T_b\)'s into rain rates by the exponential curves as in figure 3(a) for the East and West Pacific. It is apparent that the BFC increases with increasing rain rate and the rainfall inhomogeneity parameter \(CV_R\). It is also shown that the West Pacific having more inhomogeneous rainfalls than the East Pacific needs greater BFC in retrievals as rain rates increase. The collocated observations obviously suggest that BFC can be determined given the true rain rate and \(CV_R\). However, these parameters are exactly the parameters that we want to retrieve in the first place.
Figure 4. Mean beam-filling correction factors (BFCs) as a function of rain rates ($R$) at the different intervals of the rainfall inhomogeneity parameter ($CV_R$: (a) East Pacific; (b) West Pacific).

An attempt is made to compute BFC using a parameter from radiometers, $CV_{Tb}$, based on empirical relations between the two parameters. The $CV_{Tb}$ is a quotient of the standard deviation divided by the mean of $T_b$ at 85.5 GHz (equation (4)). Both the standard deviation and the mean can contribute to the variations of $CV_{Tb}$. Radiative transfer calculations show that the $T_b$ at 85.5 GHz decreases with rain rate due to scattering of ice in rain columns. Saturation reaches at the heavy rain end. The standard deviation is expected to be smaller for stratiform rain than for convective rain. Hence the low $CV_{Tb}$ at the low rain regime may be attributed to small horizontal variations of $T_b$ at 85.5 GHz and large values in the mean $T_b$. At the high rain regime, both increased standard deviation and decreased mean contributed to the high values of $CV_{Tb}$. A scatter plot between the BFC and $CV_{Tb}$, averaged in 1 mm hour$^{-1}$ rain rate bins and three different ranges of $CV_R$, is shown in figure 5 ignoring the rain and rainfall inhomogeneity categories. The high correlations (0.89–0.9) suggest linear relationships of the form $BFC = \alpha CV_{Tb}$. Since the bias statistics may be important from a climatological perspective, the coefficient $\alpha$ has been computed by minimizing the bias as follows:

$$\alpha \approx \frac{\sum R}{\sum (CV_{Tb} \times R_E)}, \quad (6)$$

where $R$ and $R_E$ are the true and estimated rain rates; then $\alpha = 0.317$ and 0.267 for the East and West Pacific, respectively.

This study used these linear relations to correct the estimated rain rates at 19.4 GHz footprints. Figure 6 shows the BFC after the $CV_{Tb}$ correction. The BFC are decreased and the dependence on mean rain rate has been much reduced. Especially, the improvement of the correction is better represented at the higher rain rates (above $\sim 3$ mm hour$^{-1}$) where the separation of vertical rainfall structures by $CV_{Tb}$ is usually clearer. Statistics for before and after the beam-filling correction by $CV_{Tb}$ are summarized in table 1. The bias statistics seemed to be well improved for both regions. The root mean square (RMS) error and correlations are also improved for the West Pacific but slightly downgraded for the East Pacific. The beam-filling error appears to be reduced through the radiometer adjustment factor, which may be constructed by climatology of rainfall and $CV_{Tb}$. 
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Figure 5. Scatter diagrams of mean beam-filling correction factor (BFC) and mean inhomogeneity parameters, \( CV_T \). The paired data (BFC, \( CV_T \)) were obtained as a function of the mean rain rate and rainfall inhomogeneity parameters (\( CV_R \)) as in figure 4. Correlation coefficients (Corr) are also indicated.

Figure 6. Mean beam-filling correction factors (BFC) are recomputed with the true rain rates and the estimated rain rates corrected by the \( CV_T \) information through the linear relationships between BFC and \( CV_T \): (a) East Pacific; (b) West Pacific.

Table 1. Statistics between the true rain rates and the estimated rain rates before and after the beam-filling correction by the spatial variability of 85 GHz \( T_b \) (\( CV_T \)).

<table>
<thead>
<tr>
<th></th>
<th>East Pacific</th>
<th></th>
<th>West Pacific</th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>Before correction</td>
<td>After correction</td>
<td>Before correction</td>
<td>After correction</td>
</tr>
<tr>
<td>Bias</td>
<td>-1.093 (-30.4)</td>
<td>0.125 (3.2)</td>
<td>-1.038 (-35.0)</td>
<td>0.077 (2.4)</td>
</tr>
<tr>
<td>RMS</td>
<td>2.557 (71.0)</td>
<td>3.384 (86.4)</td>
<td>2.241 (75.6)</td>
<td>2.167 (68.1)</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.79</td>
<td>0.70</td>
<td>0.73</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Notes: Numbers in parentheses indicate per cent value of true mean. RMS, root mean square.
4. CV for a multichannel Bayesian retrieval

4.1 Combined use of $T_b$s and CV for a Bayesian estimation

This section investigates approaches to mitigate the rainfall inhomogeneity for multichannel retrieval algorithms by adding the spatial information of the high-frequency brightness temperature observations. In a Bayesian retrieval algorithm for rainfalls using brightness temperature observations, the state vector $h$ can be rain parameters and the measurement vector $b$ will be the set of brightness temperatures from microwave radiometers. In the case that we have an additional measurement information $s$ such as the spatial variability of the high-frequency brightness temperatures, the posterior probability may be expressed as

$$P(h|b, s) \propto P(b|h, s) \times P(h|s).$$  (7)

Since $b$ and $s$ are statistically independent parameters such that the forward problem is determined by the state vector $h$ only, the posterior may be rewritten as

$$P(h|b, s) \propto P(b|h) \times P(s|h) \times P(h).$$  (8)

As outlined in Rodgers (2000), the conditional probability, $P(b|h)$ may be modelled by a multidimensional Gaussian distribution of the difference between the observation $b$ and the modelled observation, $b_m(h)$, with the uncertainty given by the measurement and forward simulation as follows:

$$P(b|h) = \frac{1}{(2\pi)^{P/2} |C_b|^{1/2}} \exp \left\{ -\frac{1}{2} [b - b_m(h)]^T C_b^{-1} [b - b_m(h)] \right\},$$  (9)

where $P$ is the dimension of the vector $b$, $C_b$ is the error covariance matrix of the difference (or error) vector $[b - b_m(h)]$, which is associated with the instrumental noise and forward simulation error, and the determinant; the transpose and the inverse of the matrix are denoted by $||$ and superscripts $^{-1}$ and $^T$, respectively. The instrumental error may be uncorrelated at the different channels but the error from forward computations may be correlated between the channels. The full covariance matrix $C$ can be computed if the prior information (database) is based on a simulation of observation fields. This study uses direct observations so that the error from the forward computations will not be an error source in the covariance matrix and the covariance matrix has then only the diagonal elements of the instrumental noises at each channel.

Similarly, $P(s|h)$, the conditional probability of $s$ given the state vector $h$, can be

$$P(s|h) = \frac{1}{(2\pi)^{Q/2} |C_s|^{1/2}} \exp \left\{ -\frac{1}{2} [s - s_m(h)]^T C_s^{-1} [s - s_m(h)] \right\},$$  (10)

where $Q$ is the dimension of the vector $s$ and $C_s$ is the covariance matrix that can be obtained from the difference between $s$ and $s_m$ from observation and model domains, respectively. This study uses the spatial information from two areas, 10.7 GHz-FOV and 19.4 GHz-FOV.

$P(h)$ is a priori probability that $h$ is the true state vector of precipitation. In the case of the prior distribution based on the PR observations, $P(h)$ for each state vector in the
prior information may be treated equally such that $P(h) = 1/N$ where $N$ is the number of profiles in databases. Naturally, the validity of this assumption will be enhanced if the number of the state vector in the prior information is sufficient enough to describe precipitation fields as found in nature.

Thus, the posterior probability will be expressed as

$$P(b|h) \times P(s|h) \times P(h) = \frac{1}{(2\pi)^{P/2}(2\pi)^{Q/2}|C_b|^{1/2}|C_s|^{1/2}} \times \exp \left\{ \frac{1}{2} \left[ (b - b_m(h))^T C_b^{-1} (b - b_m(h)) \right. 
\left. + (s - s_m(h))^T C_s^{-1} (s - s_m(h)) \right] \right\} \times \frac{1}{N}. \quad (11)$$

Now the retrieval is to evaluate the expectation for $h$ as given by

$$E(h) = \frac{\int hP(h|b,s)dh}{\int P(h|b,s)dh}. \quad (12)$$

The expectation may be evaluated as follows:

$$E(h) \approx \frac{1}{N} \sum_{t=1}^{N} h_t W(h_t), \quad (13)$$

where $t$ indicates $t$-th element in the database and $W(h_t)$ is directly determined by the posterior probability, equation (11).

### 4.2 Synthetic retrievals

In order to demonstrate how the additional information about the spatial variability of $T_b$ at 85.5 GHz ($CV_{Tb}$) can reduce the retrieval error, we compare two synthetic retrieval results: (1) retrievals with only $T_b$s and (2) retrievals with both $T_b$s and $CV_{Tb}$ at two areas (10.7 GHz-FOV and 19.4 GHz-FOV). The a priori databases constructed previously from pure observations over the East and West Pacific are used. The half of the rain profiles and $T_b$s in the database is used as the a priori database to retrieve the rest of the rain profiles. The Bayesian retrieval scheme in this study does not include any sophisticated procedures to increase retrieval performance, while focusing on the relative performance of the two synthetic simulations.

Figure 7 shows the two-dimensional histograms of the true and retrieved rain rates for the two synthetic retrieval experiments over the East and West Pacific. The first experiment using only $T_b$s (figure 7(a) and (c)) shows that all statistics (bias, RMS and correlation) indicate that the retrievals over the West Pacific are not as good as those in the East Pacific. This is due to the more non-linear radiometric signature to rainfall over the West Pacific resulting from more rainfall inhomogeneity, as shown in figure 3(b). The second experiment that includes the spatial information of high-frequency channel $T_b$ in addition to the original $T_b$s (figure 7(b) and (d)) consistently
Figure 7. Two-dimensional histograms of the true and retrieved rain rates for the two synthetic retrievals: (1) only $T_{b}$s of the Tropical Rainfall Measuring Mission (TRMM) Microwave Imager (TMI) 9 channels ($a$, $c$) and (2) adding CV $T_{b}$ information ($b$, $d$) to the first case. Frequency (occurrence) in log$_{10}$ scale given true and retrieved rain rates is indicated by colour in square box.

outperforms that of the first experiment. The results from the experiments suggest that the spatial variability of $T_{b}$ at 85.5 GHz may be a parameter to enhance the retrieval performance. However, it should be mentioned that the spatial information of the $T_{b}$ is only applicable to the *a priori* information, which is built on observation precipitation fields or accurately simulated precipitation fields. The database from Shin and Kummerow (2003) may be an example that can incorporate the CV $T_{b}$ information in the retrievals as long as the simulate database resembles the observation field closely.
5. Conclusions

It is well known that the combined effect of the non-linear relation between microwave brightness temperatures and rain rates and the satellite footprints which are greater than the spatial scale of rainfall variability causes a major retrieval error called the beam-filling error. Using the collocated PR and TMI data and radiative transfer computations with simulated precipitation profiles (figure 3), the beam-filling error has been quantified showing the dependency on the mean rain rate and the rainfall inhomogeneity parameters CV_R (figure 4). It is also found that the beam-filling error turns out to be significantly different for the two different climate regimes or possibly two different precipitation systems where different horizontal and vertical precipitation structures exist. This fact suggests the BFC obtained from a specific region or a cloud model may not be applied to a global correction. A single constant as used in some algorithms, of course, should not be a solution for the beam-filling correction.

This study investigates the relationship between the BFC and the spatial information derived from T_b at 85.5 GHz, CV_Tb. It is observed that the mean BFC and CV_Tb are linearly correlated and apparently the relation is independent on the mean rain rate. The linear relationship suggests CV_Tb can be a parameter that modulates the beam-filling error, and then the beam-filling correction by the radiometer parameter CV_Tb has been successfully demonstrated for the East and West Pacific. The linear relationship was built on the TRMM TMI and PR observations such that the BFC is only applicable to the TMI radiometer. The methodology, however, can be applied to different radiometers by using simulated rain fields. The simulation of rain fields is briefly introduced in §4 and the details of the simulation method can be referred to in Shin and Kummerow (2003). Once the simulated rain fields, which are consistent with PR and TMI observations, are obtained, the relationship between the BFC and the CV_Tb can be obtained in a parametric way that fits into different radiometers (but with high-frequency channels).

Moreover, the CV_Tb is used to reduce the retrieval error in the Bayesian-type multichannel retrieval scheme. The CV_Tb at two different areas are incorporated with the T_b vector for the Bayesian inversion. Using the synthetic retrievals, this study demonstrated the CV_Tb information improved the rainfall retrieval accuracy by reducing the effect of the rainfall inhomogeneity, although the mitigation of the retrieval error is moderate because the multichannel has already quite accurate information.

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References


WANG, S.A., 1995, Modeling the beamfilling correction for microwave retrieval of oceanic rainfall. PhD thesis, Texas A&M University, College Station, TX.

