Moist adjoint-based forecast sensitivities for a heavy snowfall event over the Korean Peninsula on 4–5 March 2004

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[1] Adjoint sensitivity analyses are applied to a heavy snowfall event on the Korean Peninsula using the MM5 Adjoint Modeling System. To evaluate the effects of initial conditions on forecast error, adjoint integrations are performed using forecast error as a response function. Initial adjoint sensitivities are located in the middle to lower troposphere with horizontally isolated upshear-tilted structures in the upstream regions (i.e., southern Mongolia). In addition, due to the effect of moist physics in adjoint model integration, vertically striped structures in the lower troposphere are also detected in the southern sea of the Korean Peninsula. Both initial adjoint sensitivity structures match the individual singular vector structures. The new 36-h forecast using the initial condition perturbed with scaled adjoint sensitivities (i.e., key analysis error) shows much improvement in the intensity and location of the surface cyclone as well as the upper trough, and reduces 42.9% of the forecast error compared to the control forecast. In addition, the forecasts using the initial condition perturbed by the key analysis error deviate slightly from the observations than the forecasts using the control analysis in the first 12 h and are much closer to the observations than the control forecasts in the later 12 h. The linear and nonlinear evolutions of temperature perturbations over the large adjoint sensitivity regions show similar growth rates and spatial distributions. Additional experiments with and without specific moist physics for linear model integrations show that disagreement of moist physics in the nonlinear and linear model integrations degrades the linearity.


1. Introduction

[2] Despite the advances in the quality of numerical weather forecasts of extratropical cyclones, notable forecast “busts” still occur. These forecast bust may be caused by the deficiencies in the numerical weather prediction models and their assimilation schemes, or due to deficiencies in the initial conditions. Exceptional cases, such as a heavy snowfall event on 4–5 March 2004 on the Korean Peninsula, could be more susceptible to these types of deficiencies. This event affected the central part of the Korean Peninsula with a maximum snowfall up to 50 cm, which is a record-breaking daily snowfall amount in March.

[3] As a method to detect deficiencies in the initial condition, adjoint sensitivity has been used for various oceanic and meteorological applications, including sensitivity analysis, optimization problems, data assimilation, parameter estimation, stability analysis, and targeted observations [Errico, 1997]. In contrast to traditional sensitivity analysis (e.g., forward sensitivity), which calculates the response of a forecast aspect by integrations of the nonlinear model (NLM) with perturbed input, the adjoint sensitivity provides gradient (i.e., sensitivity) of some forecast aspect with respect to the initial condition by a single backward integration of the adjoint model (ADJM). The forward sensitivity is computationally expensive because every variable on every grid can affect the forecast. To get general results, forward integrations of the NLM as the number of degrees of freedom are required.

[4] Many studies on adjoint sensitivities have been carried out since the early 1990s. Errico and Vukicevic [1992] developed a dry version of tangent linear model (TLM) and corresponding ADJM based on the fourth-generation Pennsylvania State University/National Center for Atmospheric Research (PSU/NCAR) Mesoscale Model (MM4), and applied adjoint sensitivity analysis to Alpine lee cyclogenesis and Atlantic explosive cyclogenesis. Rabier et al. [1992] studied the adjoint sensitivity for ideal cyclogenesis cases using a global primitive model, and successfully applied an adjoint of nonlinear normal mode initialization technique to remove the influence of gravity waves on sensitivity fields. Langland et al. [1995] studied the physical development of an idealized extratropical cyclone with the adjoint sensitivity, and found that higher sea surface temperature on the warm sector of a cyclone significantly enhances baroclinicity and thermal advection in the lower troposphere. Kleist and Morgan [2005a] studied structure
and time evolution of adjoint sensitivity of a real extratropical cyclone case with detailed physical interpretations. For weather events in the East Asia region, Lee and Chang [1997] studied sensitivity for an explosive cyclone case in the East Sea (Sea of Japan), and Wang and Gao [2003, 2006] studied the unresolved meso-$\beta$-scale vortex that is a main disturbance system in the Mei-yu front of China.

[5] On the basis of the idea that forecast errors are mainly due to initial condition uncertainty, the adjoint sensitivity can be used to reduce forecast error by adding perturbation derived from the adjoint sensitivity to the initial condition. Rabier et al. [1996] corrected one-tenth of 2-day forecast error based on the adjoint sensitivity of short-range forecast error to the initial condition, and suggested that the adjoint sensitivity of short-range forecasts can provide useful information to medium-range forecasts. Klinker et al. [1998] estimated key analysis error with an iterative procedure of adjoint calculation, and suggested that key analysis error is useful because evolution of key analysis errors closely resembles forecast error. Zou et al. [1998] showed that medium-range (5 day) forecast of East Coast oceanic cyclone can be improved using 12-h forecast sensitivity using the MM5 Adjoint Modeling System. Hello et al. [2000] perturbed observations adjacent to maximum sensitivity regions to improve mean sea level pressure (MSLP) forecast. Langland et al. [2002] reduced 72% of forecast error with two iterations and improved the intensity and location of a poorly forecasted U.S. East Coast cyclone on 25 January 2000 with the Navy Operational Global Atmospheric Prediction System. For the same case, Kleist and Morgan [2005b] reduced 46% of forecast error with 12 iterations, and by better representation of dynamic features, the corresponding precipitation forecast was also improved. Laroche et al. [2002] applied adjoint sensitivity analysis to a poor 3-day winter storm forecast of the Global Environmental Multiscale model. Recently, there have been some studies concerning the reality of key analysis error compared to observations [Isaksen et al., 2005; Caron et al., 2007a, 2007b]. For tropical cyclone applications, Wu et al. [2007] used midtropospheric averaged wind (i.e., steering flow) of typhoons as response functions of the adjoint sensitivities, and diagnosed initial sensitivities for those response functions.

[6] Compared to aforementioned studies that used dry (moist or dry) physics for ADJM (NLM) integrations, fewer studies used the same moist physics for both NLM and ADJM integrations. Zhu and Navon [1998] studied the sensitivity of 1-day forecast error for an Indian summer monsoon case, using the moist physics (i.e., large scale precipitation and convection) ADJM of the Florida State University (FSU) Global Spectral Model (GSM). Ancell and Mass [2006] used large scale precipitation and Kuo cumulus parameterization scheme for NLM and ADJM integrations to calculate adjoint sensitivities, and showed that structure, growth rates, and tangent linear accuracy of adjoint sensitivities vary significantly depending on the horizontal and vertical resolutions of the model. Using the same moist physics in NLM and ADJM integrations, Ancell and Mass [2008] further investigated the variability of adjoint sensitivity with respect to model physics and basic state trajectory. Zhong et al. [2007] used large scale precipitation and Grell cumulus convection scheme for NLM and ADJM integrations to study the adjoint sensitivity of mesoscale low in the Mei-yu front. The same moist physics (i.e., large scale precipitation and Grell convective cumulus scheme) are also used in NLM and ADJM integrations to calculate adjoint sensitivities for Typhoon Rusa [Kim and Jung, 2006]. Despite the use of certain types of moist physics for adjoint sensitivity studies, adjoint sensitivities using grid-resolvable precipitation parameterization (i.e., explicit moisture scheme) as the moist physics for both NLM and ADJM integrations have not been explored yet.

[7] Compared to other studies, this study focuses on the adjoint sensitivities and forecast error reduction for a heavy snowfall event on the Korean Peninsula using the explicit moisture and Grell convective cumulus scheme for both NLM and ADJM integrations. This snowfall case is an very rare occurrence which has not been previously studied using adjoint sensitivities. Cho et al. [2004] showed that the heavy snowfall event on 4–5 March 2004 is mainly associated with the surface cyclone and upper trough development. Therefore, the forecast failure in predicting the intensity and location of the surface cyclone may be one of the factors that contributed to the relatively incorrect forecast (i.e., around 5–10 mm of rainfall forecasts compared to 50 cm of snowfall observations) for this snowfall event. Furthermore, Hong [2004] noticed that the heavy rainfall mechanisms on the Korean Peninsula are different from those in the United States. Therefore a high-impact weather event, such as heavy snowfall in Korea, may show different characteristics of adjoint sensitivities compared to those found in other places around the globe. Because the heavy snowfall case may be affected by moisture processes over the sea surrounding the Korean Peninsula, the explicit moisture and Grell cumulus convection scheme are used for both NLM and ADJM integrations.

[8] In this study, the detailed structures and evolutions of adjoint sensitivities for the case are investigated, and the forecast error reduction method using the adjoint sensitivity is used to improve the forecast state. Moreover the sensitivity of the tangent linearity to the use of the specific moist physics for ADJM integrations is discussed. Model configuration and mathematical formulations on adjoint sensitivity and forecast error reduction are described in section 2. Results including general features of the case, adjoint sensitivity characteristics, comparison with singular vectors (SVs), forecast error reduction and comparison with observations, and tangent linear validation are presented in section 3. Section 4 contains the summary and discussion.

2. Model and Methodology

2.1. Model and Physical Processes

[9] To calculate adjoint sensitivities, this study uses the MM5 Adjoint Modeling System [Zou et al., 1997]. The state vector \( x \) for this modeling system is \( \{ U, V, W, T, Q_v, PP, \} \), where \( U, V \), and \( W \) are three-dimensional wind components, \( T \) is temperature, \( Q_v \) is water vapor mixing ratio, and \( PP \) is pressure perturbation. The model is centered at \( 40^\circ N \) and \( 113^\circ E \) with a 45 km horizontal grid spacing in a \( 90 \times 90 \) domain and 14 evenly spaced sigma levels in the vertical from the surface to 50 hPa with Lambert Conformal map projection. The model initial and lateral boundary conditions are from the European Centre for Medium-
Range Weather Forecasts (ECMWF) operational analysis data ($1^\circ \times 1^\circ$ global grid). The Optimum Interpolation Sea Surface Temperature (OISST) Version 2 [Reynolds et al., 2002] is used for the lower boundary condition over the ocean.

[10] Physical parameterizations used for the nonlinear basic state integrations include the horizontal and vertical diffusion, dry convective adjustment, bulk aerodynamic formulation of the planetary boundary layer, a simple radiational cooling scheme, Grell convective scheme, and explicit treatment of cloud water, rain, snow, and ice. The nonlinear basic state is updated at every time step ($\Delta t = 120$ s). The same physical parameterizations (i.e., full moist physics) are used in the ADJM integration. The NLM integration is conducted over a period of 36 h, from 1800 UTC 3 March to 0600 UTC 5 March 2004.

2.2. Adjoint-Based Forecast Sensitivity

[11] Nonlinear numerical model can be expressed as $x_f = N(x_i)$, where $x_i$ and $x_f$ are state vectors at the initial and final (i.e., verification) times, and $N$ is a NLM. Using Taylor expansion, variation of the final state $\Delta x_f$ can be derived from the first derivative of $N$,

$$\Delta x_f \approx \delta x_f = \left. \frac{dN}{dx} \right|_{x=x_i} \delta x_i = M \delta x_i,$$  

(1)

where $M$ is a TLM of $N$.

[12] The forecast aspect or response function $R$ is a function of a state vector at the verification time, $R = f(x_i)$, which is differentiable to the state vector. The variation of $R$ at the verification time is derived from Taylor expansion as follows:

$$\Delta R \approx \delta R = \left. \frac{dR}{dx} \right|_{x=x_f} \delta x_f,$$  

(2)

By assuming linearity, $\delta x_f$ can be expressed by $M \delta x_i$. The variation of $R$ is then

$$\Delta R \approx \delta R = \left( M^T \right) \delta x_i,$$  

(3)

where the adjoint relationship is used to deduce (4) from (3). Because the variation of $R$ is defined at the initial time as

$$\Delta R \approx \delta R = \left. \frac{dR}{dx} \right|_{x=x_i} \delta x_i,$$  

(5)

$$\frac{dR}{dx_i} = M^T \frac{dR}{dx_f}$$  

(6)

is deduced by equating the right-hand sides of (4) and (5).

[14] Therefore, the sensitivity of the response function $R$ to the initial state can be obtained from a backward integration of an ADJM with the gradient of $R$ to the final state as an input.

2.3. Correction of Initial Condition Uncertainties by Adjoint-Based Forecast Sensitivities

[15] Rabier et al. [1996] suggested that the adjoint method can be used to indicate the sensitivity of forecast errors to the initial condition. To this end, forecast error measured in dry total energy (TE) at the verification time in the vicinity of the Korean Peninsula is chosen for a response function as

$$R = \frac{1}{2} (Pe_f, CP_{e_f}),$$  

(7)

where $e_f$ is the MM5 forecast error at 0600 UTC 5 March 2004 (i.e., the deviation of the 36-h MM5 forecast state from the ECMWF analysis), $P$ is a local projection matrix, which constrains the response function to a box surrounding the model predicted extratropical cyclone over the Korean Peninsula, and $C$ is the dry TE norm matrix defined by Zou et al. [1997]. The TE norm has been used for many predictability studies [Rabier et al., 1996; Gelaro et al., 1998; Klinker et al., 1998; Langland et al., 2002; Kleist and Morgan, 2005b; Kim and Jung, 2006, 2009a, 2009b] because the TE norm is suggested as a first-order approximation of analysis error covariance matrix by Molteni et al. [1996] and Palmer et al. [1998]. Detailed formulation of the response function in (7) is as

$$R = \int \int \int \int \int \int \frac{1}{2} \left[ U''^2 + V''^2 + W''^2 + \left( \frac{g}{\overline{\theta}} \right)^2 \theta''^2 + \left( \frac{1}{\overline{\rho c_p}} \right)^2 PP''^2 \right]$$  

(8)

where $U'$, $V'$, $W'$, $\theta'$, $PP'$ are forecast errors of zonal, meridional, vertical winds, potential temperature, pressure perturbation, $\overline{N}$, $\overline{\theta}$, $\overline{\rho}$, $c_p$ are reference values of Brunt-Väisälä frequency, potential temperature, density, speed of sound, and $\sigma$, $\sigma$ are horizontal domain and vertical coordinate, respectively.

[16] If there is no model error, the adjoint sensitivity of the forecast error to the initial conditions may be used to identify those analysis errors that are specifically associated with the forecast error, and a more improved forecast may be obtained using this information. To this purpose, a perturbation ($\delta x_i^{(1)}$) is generated from the adjoint sensitivity, and the modified initial condition ($x_{\text{mod}}^{(1)}$) can be derived as

$$\delta x_i^{(1)} = \alpha C^{-1} \frac{dR}{dx_i},$$  

(9)

$$x_{\text{mod}}^{(1)} = x_i - \delta x_i^{(1)}$$

where $\alpha$ is a constant scaling factor that is determined by the ratio between the energy norm forecast error at the final
time and the energy-weighted adjoint sensitivity at the initial time [Langland et al., 2002] as

$$\alpha = \frac{1}{2} \left( \langle c_i, C e_i \rangle \right) \left( \frac{\partial R}{\partial X_i} \right) \left( C^{-1} \right) \left( \frac{\partial R}{\partial X_i} \right).$$

The energy-weighted adjoint sensitivity in the denominator of (10) combines the adjoint sensitivity to each model variable with different units into a single adjoint sensitivity field with an energy unit (i.e., joules per kilogram). The weighting $\alpha$ is determined not to overcorrect the analysis. The key analysis error and optimal initial condition can be estimated by the iterative procedure [e.g., Klinker et al., 1998; Kim et al., 2008] as

$$\delta x_{\text{key}} = \delta x_{1}^{(1)} + \delta x_{1}^{(2)} + \cdots + \delta x_{1}^{(n)}$$

$$x_{\text{opt}} = x_{i} - \delta x_{\text{key}},$$

where $\delta x_{\text{key}}$ is the key analysis error and $x_{\text{opt}}$ is the optimal initial condition. In the iterative procedure, deviation between the ECMWF analysis and a new 36-h forecast state integrated from $x_{\text{mod}}^{(1)}$ becomes a new response function, and from this response function, new adjoint sensitivity and associated perturbations $\delta x_{1}^{(2)}$ can be calculated (Figure 1). Isaksen et al. [2005] and Caron et al. [2007a] showed that the optimal iteration number for the key analysis error calculations is three to four because increasing the iteration numbers produce large and unrealistic initial corrections and, as a result, amplify noise in the initial perturbation. In this sense, the iteration in (11) is performed three times. If the initial condition uncertainty is properly estimated with the key analysis error, the forecast started from the optimal initial condition would be better than the control forecast started from the analysis.

2.4. Key Analysis Error Verification of Observations

Klinker et al. [1998] said that key analysis error does not necessarily represent the "real" analysis error but part of the real analysis error. Recently, there have been studies on whether key analysis error represents the real analysis error. Isaksen et al. [2005] found that during the first (later) 12 h, the control forecast initialized with the analysis is closer (further away) to observation than the forecast started from the initial condition perturbed by key analysis error.

Figure 1. Schematic of the iterative procedure for calculating the key analysis error and optimal initial condition.

Figure 2. Thirty-six-hour (1800 UTC 3 March to 0600 UTC 5 March 2004) accumulated liquid equivalent precipitation (contour and shading intervals of 10 mm) observed at Korea Meteorological Administration (KMA) weather station.
Caron et al. [2007a] also found that the initial condition corrected with key analysis error shows forecast error reduction but is greatly out of balance and far away from observations. To compare the modified initial condition from the key analysis error calculation in section 2.3 and subsequent forecasts with observations, the quadratic operator to measure the difference between a model state ($x$) and an observation ($y$) vector is defined as

$$J_o(x) = (y - Hx)^T R^{-1} (y - Hx),$$

where $H$ is the observation operator that projects the model state $x$ from the grid space to the observational space, and $R$ is the observation error covariance matrix. The radiosonde observations via the Global Telecommunication System (GTS) and observation error covariance matrix from Barker et al. [2004] are used to calculate (12).

The difference between $J_o(x)$ using the modified initial condition of each iterative procedure and $J_o(x)$ using the ECMWF analysis is calculated as

$$\Delta J_o = \frac{J_o(x^{(n)}) - J_o(x_i)}{J_o(x_i)},$$

where $J_o(x_i)$ is the departure between the observation and the control forecast with ECMWF analysis as an input, and $J_o(x^{(n)})$ is the departure between the observation and the forecast with the modified initial condition by $n$th iteration as an input.

### 3. Results

#### 3.1. Case Description

A heavy snowfall event on 4–5 March 2004 affected the central part of the Korean Peninsula with maximum snowfall up to 50 cm (Figure 2). Because the normal value of monthly precipitation in March is around 60 mm (mostly rainfall) in the central part of the Korean Peninsula, 50 cm of snowfall in 2 days is a record-breaking amount and very rare. MSLP and 500-hPa geopotential height of the ECMWF analysis are shown in Figure 3. At 1800 UTC 3 March 2004, the Siberian high is located in northwestern Mongolia, and a migratory high is located in the southern part of China (Figure 3a). Between the two high systems, low-pressure regions are formed in northern China. At 0600 UTC 4 March, the migratory high advances eastward and the surface cyclone is located in eastern China (with central pressure of 1008 hPa) (Figure 3b). At 1800 UTC 4 March, the Siberian high moves southward and strengthens (Figure 3c). At 0600 UTC 5 March, the Siberian high is located in the center of Mongolia, and the migratory high moves eastward (out of domain). The surface cyclone is located in the southern Korean Peninsula with central pressure of 1009 hPa (Figure 3d). In the 500-hPa geopotential
height field, a weak and broad trough is initially located above the Korean Peninsula (Figure 3a), and later, a new shortwave trough is formed in the upstream regions of the surface cyclone (Figures 3b and 3c). As the new trough advances eastward, the surface cyclone moves eastward (Figure 3d).

3.2. Control Experiment

[21] In the control experiment, the 36-h control forecast is integrated from the initial condition of ECMWF analysis. In the 12-h forecast, the surface cyclone in northern China is weaker than in the analysis (Figure 4b). While the migratory high in the analysis (Figure 3b) moves above the Yellow Sea, the migratory high in the forecast (Figure 4b) is still located in southern China. In the 24-h forecast, the control forecast simulates the broader area and deeper center of lows (Figure 4c), compared to the analysis. At this time, the migratory high above the Yellow Sea is not simulated well, and the Siberian high is extended to southern China (Figure 4c). In addition, the 500-hPa trough above China is more distinct than in the analysis (Figure 4c). In the 36-h forecast, the surface cyclone passes the Korean Peninsula with 3-hPa deeper central pressure compared to the analysis, and the 500-hPa trough is located above the Yellow Sea (Figure 4d). As a whole, the control forecasts simulate a more organized fast-moving surface cyclone with deeper central pressure compared to the analysis, but do not simulate expansion and movement of the surface anticyclones well.

3.3. Adjoint-Based Forecast Sensitivity Structures and Effect of Moist Physics

[22] The maximum of the forecast error is located in the upper troposphere (350 hPa) with the secondary peak in the lower boundary (Figure 5b). A total of 46.6% of the dry TE of the forecast error is attributed to the error in meridional wind, 28.0% to the temperature error, 25.0% to the zonal wind error, and the remaining 0.4% to the errors in vertical wind and pressure perturbations.

[23] Evolutions of the adjoint sensitivities to temperature ($\partial R/\partial T$) on 750-hPa level are shown in Figure 6. Note that as the adjoint sensitivities are integrated backward with respect to time, the ADJM is integrated from 0600 UTC 5 March (Figure 6d) to 1800 UTC 3 March (Figure 6a). Adjoint sensitivity of $R$ at 0600 UTC 5 March is evolved to upstream regions during backward integration, and evolves to regions of maximum sensitivities at 1800 UTC 3 March. Because the magnitude of sensitivities amplifies during its backward evolution, given the same magnitude of perturbations, perturbations at 1800 UTC 3 March have more impact to the response function than those at the later time.

[24] Initial sensitivity is horizontally isolated in upstream regions (Figure 6a) and vertically located in the lower to midtroposphere (Figures 5a and 7a). Adjoint sensitivities show westward tilted baroclinic structures at the initial and
subsequent times, and finally become vertical at the verification time (Figure 7). While maxima of the adjoint sensitivities are located in the lower to midtroposphere under the upper level trough (Figures 7a–7c), maxima of the forecast error are located in the upper troposphere (Figure 7d and Figure 5b). Horizontal and vertical characteristics of the adjoint sensitivities in the upstream regions in Figure 6a are consistent with those in previous studies.

Different from the previous studies, the horizontal locations of maximum sensitivities vary at each vertical level for this case. In the lower troposphere (e.g., 850 hPa), adjoint sensitivities at the initial time are located in the southern sea of the Korean Peninsula, and the sensitive regions correspond well with the 850-hPa thermal trough (Figure 8a). Vertical cross sections through the sensitivity maxima on the 850-hPa level in Figure 8 show that the sensitivities near the Korean Peninsula have quite vertical structures at the initial time (Figure 9a), which is different from the typical upshear tilted baroclinic structures in southern Mongolia. During the backward evolution of ADJM from 0600 UTC 5 March, these low-level sensitivities show similar vertical structures except slightly westward tilted structures at 1800 UTC 4 March (Figure 9). The magnitudes of the low-level sensitivities amplify roughly 6 to 2 times for the first 12 to 24 h of backward integration of ADJM from 0600 UTC 5 March and remain similar after then (Figure 9). These low-level vertical sensitivities may be associated with convection, indicated as low-level clouds in Figure 10. Because the warm advection over the ocean made the lower atmosphere unstable in this case [Jung et al., 2005], the warm advection over the ocean may cause the convection in the lower level. The magnitude of sensitivities at the lower troposphere (e.g., 850 hPa; Figure 8a) is larger than that at the midtroposphere in southern Mongolia (e.g.,

Figure 5. Vertical distributions of the energy-weighted (a) adjoint sensitivity (J kg$^{-1}$) at the initial time and (b) forecast error (J kg$^{-1}$) at the verification time.

Figure 6. Adjoint sensitivity of forecast error to temperature (J kg$^{-1}$ K$^{-1}$, shaded line (shaded and dashed line) for positive (negative) values, shading interval varies) and temperature (solid line, contour interval of 5 K) on 750 hPa level at (a) 1800 UTC 3 March, (b) 0600 UTC 4 March, (c) 1800 UTC 4 March, and (d) 0600 UTC 5 March 2004.
500 hPa) (not shown), consistent with maximum sensitivities in the lower troposphere in Figure 5a.

To see the effect of using specific moist linear physics on the sensitivity structures, adjoint sensitivity structures with dry linear physics, large-scale precipitation as the moist linear physics, and the full moist linear physics (i.e., explicit moisture and Grell convective scheme), with full moist basic state are compared (Figure 11). While the dry linear physics do not capture the small-scale vertical structures in the southern sea of the Korean Peninsula (Figure 11a), the large-scale precipitation in ADJM integration indicates that only the southern sea of the Korean Peninsula is sensitive (Figure 11b). The full moist linear physics denote that both upstream and the southern sea of the Korean Peninsula are sensitive (Figure 11c). This implies that the full moist physics in ADJM integration are necessary to fully reveal all the sensitivity structures.

3.4. Comparison With Singular Vector Structures

The adjoint sensitivity is the linear combination of SVs that are the fastest growing perturbations during a specified time period (i.e., the optimization interval) for a given norm and basic state [e.g., Gelaro et al., 1998; Kim et al., 2004]. As described in section 3.3, the vertically integrated, energy-weighted adjoint sensitivity has two distinct horizontal locations of maxima: Mongolia and the southern sea of the Korean Peninsula (Figure 11c). In contrast, the composite of vertically integrated energy distribution from the first to tenth SV has a maximum region on the southern sea of the Korean Peninsula (Figure 12a). The first SV to third SV show the largest magnitude in the same region of Figure 12a (not shown), but the fourth SV shows the maximum in Mongolia (Figure 12b), which implies that the adjoint sensitivities are indeed explained by the linear combinations of the SVs. The dominant growth rate of the first SV compared to other SV members (Figure 12c) is due to large linear growth associated with the moist physics used for the ADJM integrations, as indicated by Coutinho et al. [2004] and Kim and Jung [2009b].

3.5. Forecast Error Reduction by Adjoint-Based Forecast Sensitivities

In this section, the forecast error reduction method in section 2.3 is used to reduce errors in the initial condition, which leads to the reduction of errors in forecasts. The scaling factor $\alpha$ is calculated as $6.18 \times 10^{-3}$ following equation (10). Magnitudes of perturbations after the first iteration ($|\Delta x^{(1)}|$) are less than 0.7 m s$^{-1}$, 0.6 m s$^{-1}$, and 0.4 K for zonal and meridional wind components and temperature, which are smaller than those of the basic state vector. The location of the surface low in the 36-h forecast using the modified initial condition from the first iteration.
(Figure 13c) is moved southwestward compared to the control forecast (Figure 13b), consistent with the location of the surface low in the analysis (Figure 13a). Moreover, the dry TE-weighted forecast error decreases around 21.7% after the first iteration (Figure 14). Much of reduction is found in the upper troposphere where the forecast error is the largest (Figure 15). The dry energy-weighted forecast error decreases as the iterations in Figure 1 continue, and a total of 42.9% of the forecast error is reduced after three iterations (Figure 14). By the iterative procedures, the intensity and location of the predicted surface cyclone and 500-hPa trough become close to those of the analysis (Figure 13). The magnitude of the estimated key analysis error (e.g., $j\delta x_{key} = j\delta x^{(1)} + j\delta x^{(2)} + j\delta x^{(3)}$) is less than 0.8 m s$^{-1}$, s$^{-1}$, 0.8 m s$^{-1}$, and 0.5 K for zonal and meridional wind perturbations and temperature perturbations. Despite the improvement of the 36-h forecast, the improvement is limited to the verification region where the response function is defined. For example, the extended Siberian high over southern China is not improved in the modified forecast compared to the control forecast (not shown).

Additional experiments perturbing the initial conditions with the scaled adjoint sensitivities located in southern Mongolia (i.e., large-scale sensitivity regions) and the southern sea of the Korean Peninsula (i.e., small-scale sensitivity regions) separately were conducted to see the influence of the large- and small-scale sensitivities on the forecast error. In spite of larger sensitivity magnitude in the small-scale sensitivity area (Figure 8), perturbing the
small-scale sensitivity regions has a smaller impact on the forecast error because the magnitude of sensitivities is larger in very small area, confined horizontally and vertically. In contrast, perturbing the large-scale sensitivity regions has a larger impact (roughly 12 times larger than the impact of the small-scale sensitivities) on the response function (i.e., forecast error), because the large-scale sensitivities are located over broad horizontal and vertical area in spite of a smaller sensitivity magnitude at a specific level as shown in Figure 6.

3.6. Comparison of the Forecasts With Modified Initial Condition of Observations

To investigate how the forecasts using initial condition modified by the key analysis error are close to the observations, equation (13) is calculated at the initial and subsequent forecast times. Compared to the control forecasts initialized with the ECMWF analysis, the forecasts with the modified initial conditions from one to three iterations deviate slightly from observations for the first 6 h, but are much closer to the observations for the later 18-h and 30-h forecasts (Figure 17). Even though the control forecasts initialized with the ECMWF analysis is closer to the observations for the short forecast range, $\Delta f_o$ in equation (13) is less than 1%. This implies that the key analysis error is still useful in reducing forecast error, since

Figure 9. Vertical cross sections of adjoint sensitivity of forecast error to temperature (J kg$^{-1}$ K$^{-1}$, shaded line (shaded and dashed line) for positive (negative) values, shading interval varies), potential temperature (solid line, contour interval of 3 K), and potential vorticity (bold line, contour interval of 0.5 PVU) along the line of (a) EE' at 1800 UTC 3 March in Figure 8a; (b) FF' at 0600 UTC 4 March in Figure 8b; (c) GG' at 1800 UTC 4 March in Figure 8c; and (d) HH' at 0600 UTC 5 March 2004 in Figure 8d. The ordinate represents the vertical level (hPa).

Figure 10. The infrared image from KMA at 1801 UTC 3 March 2004.
its evolution could represent a great part of rapidly evolving forecast error. Note that the disagreement at the 6-h forecast increases very slightly from the first to third iterations, which implies that the key analysis error from a large number of iterations may produce overly large, unbalanced perturbations, as mentioned by Caron et al. [2007a]. This inconsistency between key analysis error and real analysis error may be due to perfect model assumption and simplifications in linearization of NLM.

3.7. Evaluation of Linearity

To verify the linearity assumption on which the adjoint sensitivity analysis is based, the ratio of 36-h non-linearly and linearly evolved temperature perturbation magnitudes is compared in Table 1. The ratio is defined as the non-linearly over linearly evolved temperature perturbation magnitude, same as in the work of Zou et al. [1997]. Because tangent linearity depends on the perturbation magnitude, initial temperature perturbations of 2 K (i.e., typical analysis error magnitude), 1 K, and 0.5 K are chosen to calculate the ratio. Magnitudes less than 0.5 K are not tested, since behavior of finite perturbations has more practical interests than that of infinitesimal perturbations, as mentioned by Errico and Raeder [1999]. The temperature perturbations were put to the regions where the magnitude of the sensitivity exceeds 20% of the maximum magnitude in the sensitivity field at 500 hPa (centered in southern Mongolia) and 850 hPa (centered in the southern sea of the Korean Peninsula), similar to Ancell and Mass [2006]. In terms of ratio, the linearity holds generally well with temperature perturbations of 0.5 and 1 K in the mid troposphere of the upstream region, except temperature and water vapor mixing ratio variables for 0.5-K temperature perturbations (Table 1). The temperature perturbations in the southern sea of the Korean Peninsula show a generally better agreement as the magnitude of the temperature perturbations becomes smaller. The ratios smaller than one indicate that linear growth is larger than nonlinear growth for the temperature perturbations in the southern sea of the Korean Peninsula, which is consistent with the large singular values of the leading SV in Figure 11d.

In addition, individual structures of the linearly and nonlinearly evolved temperature perturbations are compared. The linearly and nonlinearly evolved zonal wind components attributed to the temperature perturbations of 2 K at 500 hPa are similar (Figures 18a and 18b). Owing to

Figure 11. Mean sea level pressure (solid line, contour interval of 4 hPa) superimposed on vertically integrated, energy-weighted adjoint sensitivity with (a) dry linear physics (shaded line, shading interval of $7 \times 10^3$ J kg$^{-1}$), (b) large-scale precipitation in adjoint model (ADJM) integration (shaded line, shading interval of $5 \times 10^5$ J kg$^{-1}$), and (c) full moist physics (i.e., explicit moisture and Grell convective scheme) in ADJM integration (shaded line, shading interval of $5 \times 10^4$ J kg$^{-1}$) at the initial time.
overall large linear evolution, the linear evolution has a larger magnitude than nonlinear evolution (ratio = 0.741 in Table 1). The linearly and nonlinearly evolved zonal wind components attributed to the temperature perturbations of 2 K at 850 hPa show similar structures, except additional structures for linear evolution in southern Japan (Figures 18c and 18d). The linearly and nonlinearly evolved meridional wind, temperature, and water vapor mixing ratio components at 500 and 850 hPa show characteristics similar to the zonal wind components at the same vertical levels shown in Figure 18 (not shown).

Concerned with the linearization of moist diabatic processes, discontinuous on-off process and conditionals are indicated as a main cause to degrade tangent linearity [e.g., Vukicevic and Errico, 1993; Mahfouf, 1999; Errico and Raeder, 1999]. However, moist linearization is favorable because more realistic adjoint sensitivities can be obtained [e.g., Mahfouf, 1999; Errico and Raeder, 1999; Ehrendorfer et al., 1999; Coutinho et al., 2004]. In this study, an explicit moisture scheme is used as a microphysical parameterization (grid-resolvable precipitation), which is more complex (i.e., highly nonlinear) than the large-scale precipitation scheme used by Mahfouf [1999] and Coutinho et al. [2004]. To investigate the moist effect on the linearity, we conducted an additional linearity test by using dry physics, a large-scale precipitation scheme, and full moist physics (i.e., explicit moisture and Grell convective scheme) for TLM integrations. The nonlinearly and linearly evolved perturbations using the dry physics and large-scale precipitation for TLM integrations show worse agreement compared to those using full moist linear physics. The worse agreement in using the simplified moist linear physics (i.e., dry physics or large-scale precipitation) may be due to disagreement of moist physics schemes between NLM and TLM integrations, as indicated by Ancell and Mass [2008].

4. Summary and Discussion

In this study, adjoint sensitivity analysis is applied to a heavy snowfall event on the Korean Peninsula using MM5 Adjoint Modeling System. A numerical experiment is performed on the domain centered in the East Asia region from 1800 UTC 3 March to 0600 UTC 5 March 2004. Compared to the analysis, the control forecast, which initialized with the ECMWF analysis, simulated a more organized fast-moving surface cyclone with deeper central pressure, and a more intense upper trough at the verification time. To investigate the effect of the initial condition uncertainty on the 36-h forecast, dry TE of the forecast error at the verification time in the regions around the surface cyclone is defined as the response function.
Adjoint sensitivities at the initial time have two distinct structures. One is similar to the typical structures of baroclinic instability, which is horizontally localized in the upstream region of southern Mongolia and vertically upshear tilted structures confined in the middle to lower troposphere. The other shows very small-scale structures horizontally located near the verification region (i.e., the southern sea of the Korean Peninsula) and vertically in the lower troposphere, which have not been reported in other studies. These small structures near the sea surface in the lower troposphere may be due to convection in the lower level, and are detected by adjoint sensitivities because the full moist physics (i.e., explicit moisture and Grell convective scheme) are used for ADJM integrations in this study. These exceptional vertical sensitivity structures in the lower troposphere near the Korean Peninsula may be associated with the geographical particularity of the Korean Peninsula surrounded by the ocean, which is more easily affected by the convection. Similar horizontally confined small sensitivities.

Figure 13. Mean sea level pressure (solid line, contour interval of 4 hPa) superimposed on 500-hPa geopotential height (dashed line, contour interval of 60 gpm) for (a) analysis, (b) control forecast, (c) modified forecast after first iteration, and (d) modified forecast after third iteration.

Figure 14. Energy-weighted forecast error for each iteration process. The number zero in the abscissa represents the control forecast.

Figure 15. (a) Vertical distributions of the energy-weighted forecast error (J kg⁻¹) and (b) difference (J kg⁻¹) between forecast errors of the control run and each modified run by the adjoint-based iterative procedures (solid circles, control; open circles, first iteration; solid squares, second iteration; and open squares, third iteration).
Activity structures have been found for adjoint sensitivity and singular vector studies using moist linear physics for extratropical and tropical cyclones [Ehrendorfer et al., 1999; Coutinho et al., 2004; Kim and Jung, 2006, 2009b]. Both sensitivity structures in the southern Mongolia and near the Korean Peninsula are consistent with individual SV structures.

Using the calculated adjoint sensitivities, the initial perturbation similar to the analysis error can be obtained, and the initial condition is modified by subtracting the initial perturbation from the analysis field. Compared to the control forecast, the forecast initialized by the modified initial condition shows better agreement with the analysis in location and intensity of the surface cyclone as well as 500-hPa trough and reduces 42.9% of the dry TE of the control forecast error. Perturbing the large-scale sensitivity regions over southern Mongolia has a larger impact on the forecast error than the small-scale sensitivity regions over the southern sea of the Korean Peninsula. Even though the forecast with initial condition perturbed by the key analysis error deviates slightly from the observations in the first 12 h and much closer in the later 12 h, as indicated by Isaksen et al. [2005] and Caron et al. [2007a, 2007b], the key analysis error is in a thermal wind-type balance from the comparison with PV perturbations, similar to Kleist and Morgan [2005b]. From these results, it is confirmed that the key analysis error deduced from the adjoint sensitivity may be a

![Figure 16](image1.png)  
**Figure 16.** Temperature perturbation (dark (light) shading for positive (negative) values) and PV perturbation (solid line (dashed line) for positive (negative) values) from the iterative procedures at (a) the initial time (shading interval of 0.2 K, contour interval of 0.02 PVU), (b) 12 h (shading interval of 0.2 K, contour interval of 0.03 PVU), (c) 24 h (shading interval of 0.3 K, contour interval of 0.04 PVU), and (d) the verification time (shading interval of 0.7 K, contour interval of 0.2 PVU).

![Figure 17](image2.png)  
**Figure 17.** Relative departure of $J_o(x)$ initialized with the modified initial conditions of one to three iterations from that initialized with the European Centre for Medium-Range Weather Forecasts analysis at each forecast time: (a) 6-h forecast (0000 UTC 4 March), (b) 18-h forecast (1200 UTC 4 March), and (c) 30-h forecast (0000 UTC 5 March). The modified initial conditions and observations are compared only at 0000 and 1200 UTC because there are few observational data at 0600 and 1800 UTC. The unit of the ordinate is percentage.
good approximation of the initial condition uncertainty, and the consequent modified forecast has less forecast error than the control forecast for this case.

[38] Because the adjoint sensitivity analysis is under the linearity assumption, linearity of the evolution is investigated to verify the reliability of the results. The linearity is evaluated by comparing the nonlinear and linear evolutions of specific perturbations. Following the suggestion of Errico and Raeder [1999], initial temperature perturbations with finite size are chosen over the locations of large adjoint sensitivities. With a little exceptions, the nonlinear-to-linear ratios of individual variables are close to one for smaller magnitudes of temperature perturbations in regions of large sensitivity, which implies that the linearity generally holds well. Overall, the nonlinear and linear evolutions show similar structures near the Korean Peninsula, but compared to the nonlinear evolutions, the linear evolutions of perturbations have additional structures in southern Japan. Additional linearity tests with dry physics and large-scale precipitation in TLM integrations show much worse agreement in terms of magnitude and structures, implying that disagreement of moist physics used in NLM and TLM integrations may degrade the linearity. Therefore the full moist physics (i.e., explicit moisture and Grell convective scheme) are necessary to reveal all the sensitive structures for this case.

[39] Overall, the moist adjoint-based forecast sensitivities can capture relevant sensitive structures, and the forecast error reduction method is useful in correcting the predicted intensity and location of the surface cyclone, as well as the 500-hPa trough. Since these factors are associated with the heavy snowfall event over the Korean Peninsula, this method may improve forecasts of similar events.

Table 1. Ratio of Nonlinearly Evolved Temperature Perturbation to Linearly Evolved Temperature Perturbation in the Projection Area

<table>
<thead>
<tr>
<th>Measure Variable</th>
<th>2 K</th>
<th>1 K</th>
<th>0.5 K</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Upstream, Midlevel (500 hPa)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>0.741</td>
<td>1.007</td>
<td>1.159</td>
</tr>
<tr>
<td>V</td>
<td>0.762</td>
<td>0.967</td>
<td>1.037</td>
</tr>
<tr>
<td>T</td>
<td>0.645</td>
<td>1.014</td>
<td>1.472</td>
</tr>
<tr>
<td>Q</td>
<td>0.788</td>
<td>1.283</td>
<td>1.987</td>
</tr>
<tr>
<td><strong>Over the Ocean, Low Level (850 hPa)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>0.549</td>
<td>0.754</td>
<td>0.802</td>
</tr>
<tr>
<td>V</td>
<td>0.516</td>
<td>0.726</td>
<td>0.809</td>
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<tr>
<td>T</td>
<td>0.491</td>
<td>0.690</td>
<td>0.773</td>
</tr>
<tr>
<td>Q</td>
<td>0.588</td>
<td>0.794</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The temperature perturbations over the large sensitive regions in southern Mongolia in 500 hPa and the southern sea of the Korean Peninsula in 850 hPa are tested.

Figure 18. Zonal wind component of (a) linearly evolved perturbations (contour interval of 0.5 m s\(^{-1}\)) and (b) nonlinearly evolved perturbations (contour interval of 0.5 m s\(^{-1}\)) on 500 hPa, and (c) linearly evolved perturbations (contour interval of 1.0 m s\(^{-1}\)) and (d) nonlinearly evolved perturbations (contour interval of 0.5 m s\(^{-1}\)) on 850 hPa. The box denotes a geographic region for defining a response function at 36 h.
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